ALAGAPPA UNIVERSITY

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DIRECTORATE OF DISTANCE EDUCATION

M.Sc., PHYSICS

IV –SEMESTER

34542

NUCLEAR AND PARTICLE PHYSICS

SYLLABI-BOOK MAPPING TABLE

34542 NUCLEAR AND PARTICLE PHYSICS

BLOCK I; NUCLEAR DECAY AND NUCLEAR MODELS

Syllabi	Mapping in Book
Unit I Nuclear decay- Alpha and Gamma decay Gamow's theory of Alpha decay- Gamma decay- Internal conversion- Nuclear isomerism	
Unit II Nuclear decay- Beta decay Fermi's theory of Beta decay- Kurie plots- Selection rules- Electron capture- Parity violation in Beta decay- Neutrinos- Measurement of neutrino helicity	
Unit III NUCLEAR LIQUID DROP AND COLLECTIVE MODELS Liquid Drop model- Bohr wheeler theory- Schmidt lines- Magnetic dipole moment-Electric quadrupole moment-collective model	
UNIT IV NUCLEAR SHELL MODEL Shell model- Single particle model, its validity and limitations- Rotational spectra- Magic numbers- spin-orbit coupling- Angular momentum of nuclear ground state-Magnetic moments of the shell model.	

BLOCK II: NUCLEAR FISSION, FUSION

UNIT V NUCLEAR REACTION AND MECHANISM Nuclear fission and Fusion, Nuclear reactions, reaction mechanism, compound nuclei and direct reactions: simple theory of deuteron- Tensor forces (qualitative)	
UNIT VI NUCLEAR FORCE Nature of nuclear force, form of nucleon-nucleon potential, charge independence and charge symmetry of nuclear forces- Normalization of deuteron wave functions.	
UNIT VII PARTIAL WAVE ANALYSIS Method of partial wave analysis and phase shifts- Effective range theory- n-p scattering at low energies- Yukawa's meson theory of nuclear forces.	

BLOCK III: REACTION CROSS SECTIONS AND NUCLEAR REACTORS

UNIT VIII REACTION CROSS SECTIONS Nuclear cross sections- Compound nuclear formation and breakup –Reasonance scattering cross section.	
UNIT IX NEUTRONS Interaction of neutrons with matter- Thermal neutrons- neutron cycle in thermo nuclear reactor- critical size.	
UNIT X NUCLEAR REACTORS Types of nuclear reactors- cylindrical and spherical- sub-nuclear particles (elementary Ideas only)- sources of stellar energy- controlled thermo nuclear reactions	

BLOCK IV: ELEMENTARY PARTICLES

UNIT XI FUNDAMENTAL INTERACTIONS IN NATURE

Classification of fundamental forces- Particle directory and quantum numbers (charge, spin,Parity, iso-spin, strangeness etc).

UNIT XII CLASSIFICATION OF ELEMENTARY PARTICLES

Leptons, Baryons and quarks, spin and parity assignments, isospin, strangeness

UNIT XIII GEL-MANN-NISHIJIMA RELATION

The fundamental interactions- Tranlations in space- Rotations in space- SU(2) and SU(3) groups- Charge conjugation- Parity- Gell-Mann-Nishijima formula.

UNIT XIV SYMMETRIES

Time reversal- CPT invariance- Applications of symmetry arguments to particle reactions, Parity non-conservation in weak interaction; Relativistic kinematics

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BLOCK I; NUCLEAR DECAY AND NUCLEAR MODELS

BLOCK III: REACTION CROSS SECTIONS AND NUCLEAR REACTORS

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Block I: NUCLEAR DECAY AND NUCLEAR MODELS

UNIT I NUCLEAR DECAY-ALPHA AND GAMMA DECAY

Structure

1.1 Introduction

Many of the important properties of atomic nuclei, as well as the experimental evidence for those properties are discussed. The nucleus of the atom, its atomic mass is decreased by four units and its atomic number is decreased by two units when the elements occur the natural radioactivity process, Only heavy nuclei with A>200 endure α-decay. The particle emitted from nuclei have discrete energy spectrum and comprise of several groups. Usually the most exhaustive is the group with α-particle of highest energy. In this and the following chapter we shall discuss many nuclear process and transformations which not only interesting in themselves but also provide additional information concerning the nucleus.

1.2 Objectives:

α- decay is discussed- Gamow's theory of α-decay is deliberated- the logic of internal conversion and nuclear isomerism concepts are discussed.

1.3 Gamow's theory of α-decay

Classical physics fails to elucidate α-decay. Quantum mechanics offers successful explanation of the problem of α-decay. The potential practiced by an α-particle when it approaches a nucleus is as shown in Fig.1.1

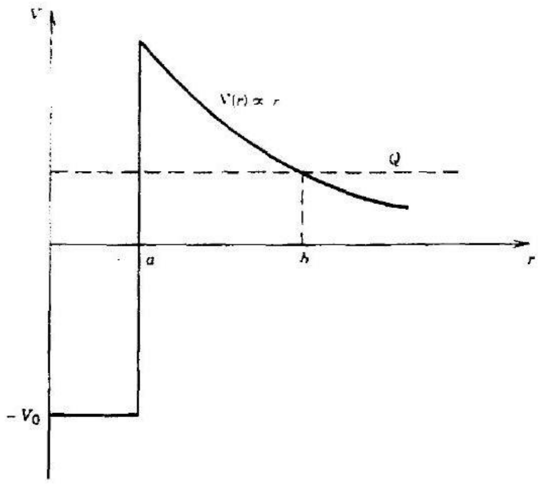


Fig.1.1

The rise of the curve from A to B shows the increase of repulsive force as the α-particle approaches the nucleus. The height of the potential barrier is found to be 28 MeV, whereas the energy of the fastest α-particle is found to be about 4.1 MeV. This shows that α-particles are penetrating the barrier. When four nuclei, two protons and two neutrons of sufficient energy combine near the nuclear surface, they can form an α-particle. The α-particle so formed has some probability for leaking through the potential barrier. The Gamow's theory of α-decay relates the K.E. If the α particle and the height of the potential barriers with the probability of leakage.

Let us imagine a beam of α-particle moving along z-direction with a velocity v. Let the beam be incident on a potential barrier of height V_0. Let the incident particle be represented by a plane wave function $\psi = e^{ik.z}$ Let there be only one particle per unit volume in the incident beam then the incident flux= $|\psi|^2$v=v. Let E be the energy of the incident α-particles.

Consider the particles to move from left to right is shown in Fig.1.2 with E>V_0.

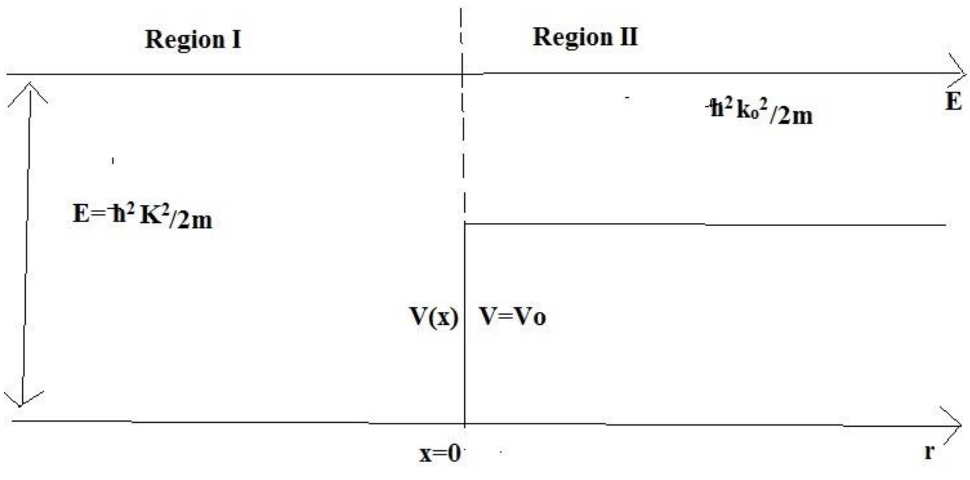


Fig.1.2

Let V=0 for x<0 (region I)

$V=V_0$ for $0 < x < \infty$ (regionII)

Let k_1 and k_2 be the wave vectors in the region I and II which are given as,

$\frac{\hbar^2 k_1^2}{2m}$ = E and $\frac{\hbar^2 k_2^2}{2m}$= ($E-V_0$)

The S.E for region I and region II are given as

$\nabla^2 u_1 + k_1^2 u_1 = 0$ ---------------------------------------(1)

$\nabla^2 u_2 + k_2^2 u_1 = 0$ --------------------------------------- (2)

The solution for equ. (1) and (2) are

$u_1 = e^{ik1x} + Re^{-k1x}$

$u_2 = Te^{ik2x}$

Where R is the reflection coefficient and T is the transmission coefficient.

Applying boundary condition namely x=0,

$u_1 = u_2$

and $\frac{du_1}{dx} = \frac{du_2}{dx}$

Applying the above conditions and solving, we get

1+R=T

$1-R=T.\frac{k_2}{k_1}$

Solving we get

$R=\frac{k_1-k_2}{k_1+k_2}$ ---(3)

And $T=\frac{2k_1}{k_1+k_2}$ -- (4)

If E is less than V_0, then R and T are given by equations (3) and (4) with the replacement of k_2 by iK where $K^2=\frac{2m}{\hbar^2}(V_0-E)$.

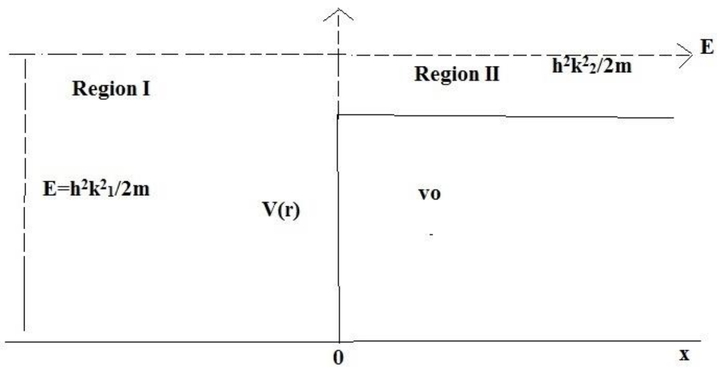


Fig.1.3

If the particles move from left to right as in Figure 1.3, R and T are given by equation (3) and (4) for $E>V_0$ when $E<V_0$ and if the particles are moving from right to left, R and T are given by equation (3) and (4), but k_1 replaced by iK.

Consider a situation as shown in figure 1.4

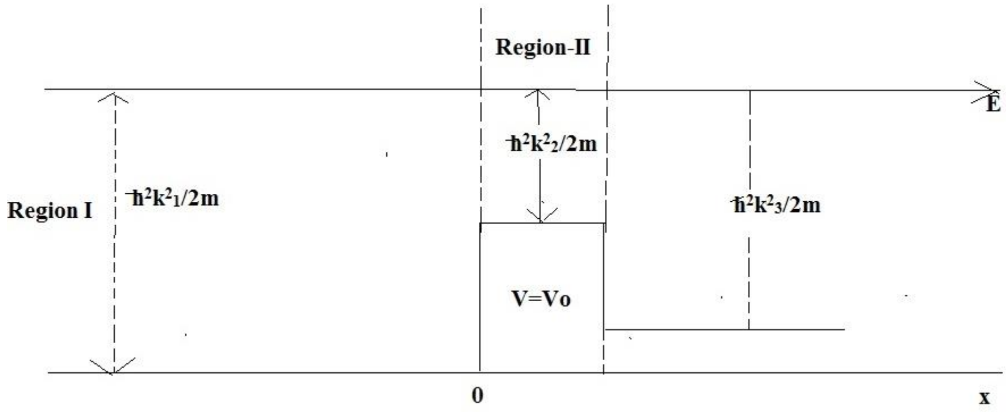


Fig.1.4

Let the incident beam undergoes multiple reflection inside the potential and gets transmitted. Hence there are many transmitted waves. The total transmission coefficient is

$T= T_{12}e^{ik_2a}T_{23}[\, 1+ R_{23}R_{21}e^{2ik_2a}+ (R_{23}R_{21}e^{2ik_2a})^2 + (\ldots\ldots)^3+----]$

Where T_{12} is the change in the amplitude of the transmitted wave, R_{23} is the change in the amplitude of the reflected wave and so on.

The barrier in the bracket $T= \frac{1}{1-R_{23}R_{21}e^{2ik2a}}$

$$R_{23} = \frac{k_2-k_3}{k_2+k_3} \;; \qquad R_{21}=\frac{k_2-k_1}{k_2+k_1}$$

Simplifying, $T= \frac{4k_1\, k_2e^{ik2a}}{(k_1+k_2)\,(k_2+k_3)}$ -------------------(5)

If $E<V_0$ equation (5) becomes

$$T= \frac{4ik_1\, ke^{-ka}}{(k_1+ik)\,(ik+k_3)}$$

The outgoing flux= $|T|^2$ v

$$= \frac{16k_1^2k^2v}{(k_1^2+k_2^2)\,(k_2^2+k_3^2)}\, e^{-2Ka}$$

The barrier penetration factor(s) is defined as the ratio of the outgoing flux to incoming flux.

If $k_1=k_2$ $\quad B=\frac{16e^{-2ka}}{(\frac{k_1}{k}+\frac{k}{k_1})^2}$

If e^{-2ka} is very small then $\frac{16}{(\frac{k_1}{k}+\frac{k}{k_1})^2}$ =1, then $B=e^{-2ka}$

But in reality the potential experienced by the α-particle is not a square well potential. It is a smooth curve consisting of a number of square well potentials of small width da.

Then $B=\int_{x1}^{x2} e^{-2ka}\, da$---------------------------- (6)

The above discussion is for the one dimensional potential. But α-particles experience a three dimensional potential. The potential function which describes the actual potential energy of the α-particles is developed by E.Igo which is given as,

$$V(r) = \frac{2.88}{r} z - e^{-1100[-r-\frac{1.173A^{2/3}}{0.574}]}$$

Where z is the atomic number of the parent nucleus. Let E_α be the energy of the α-particles and Eo be the energy of the daughter nucleus.

Then, $E=E_\alpha+E_D$

$$= \frac{p^2}{2m_\alpha}+ \frac{p^2}{2m_D}$$

$$= E_\alpha[1+\frac{m_\alpha}{m_D}]$$

$$=E_\alpha[1+\frac{4}{A}]$$

Where A is the mass number of the daughter nucleus.

The decay constant λ is related to the barrier penetration factor as $\lambda=\lambda_0 B$ where λ_0 is the decay constant in the absence of the barrier.

The wave function of the particle is given as $\psi = \frac{u_l(r)}{r} y_{l,m}(\Theta, \phi)$ where $u_l(r)$ is the radial part of the equation. The S.E. becomes

$$\frac{d^2u_l}{dr^2} + \frac{2m}{\hbar^2} K^2 u_l=0$$

Where $K^2 =\frac{2m}{\hbar^2} [V_0 - E - \frac{l(l+1)\hbar^2}{2mr^2}]$ where m represents the reduced mass.

Let u_i^+ and uf be the wave function of the α-particle inside and outside of the nucleus which are given as,

$u_i^+ = e^{iki.r}$

$u_f=e^{ikf.r}$

Then the barrier penetration factor is,

$$B= \frac{\int |\psi_f|^2 r_j{}^2 d\Omega}{\int |\psi_i|^2 r_i{}^2 d\Omega} =\frac{|u_f|^2}{|u_i{}^2|}$$ ------------------------- (7)

Equation (7) is the same as the equation (6) with a difference of using 3-dimensional wave function.

1.4 Gamma decay

The emission of electromagnetic waves is the classical equivalent of gamma emission. The periodic fluctuation in the charge and current distribution inside the nuclei set up electric and magnetic field at different points which produces an outgoing electromagnetic wave which is known as gamma radiation (γ-rays). The generation of the electromagnetic radiation is the consequence of interaction of electric and magnetic fields.

Selection rules for gamma transitions:

1. The difference in energy between the initial and final states is equal to $h\gamma$.

2. The charge is conserved.

3. The difference in the angular momentum is $lh/2\pi$.

4. Conservation of parity.

1.5 Internal conversion:

According to E.Rutherford, a nucleus is an excited state can come to the lower energy state by the emission of a γ-ray along with some energy to the electrons surrounding the nucleus. The energy of the electrons is equal to $(E_i-E_f)-E_B$ where E_B is the binding energy of the electron. As γ-ray is internally converted into an electron, the process is known as internal conversion. The electrons produce a series of mono energetic lines, the electron with lowest energy to the k-shell electron gives as $E_{ek}= (E_i-E_f)-E_k$.

Similarly, we have $E_{eL}= (E_i-E_f)-E_L$ and so on. The transition energy can be directly measured from the spectrum of conversion electrons. The emitted electron energy is independent of the emission of light quantum.

The total transition probability 'λ' from a nucleus state 'a' to the nuclear state 'b' is given as, $\lambda= \lambda_e+\lambda_\gamma$ where λ_e and λ_γ are the partial decay constants and conversion electron emission and γ-emission. The ratio λ_e/λ_γ is defined as conversion coefficient.

Conversion coefficient $\alpha=\frac{N_e}{N_\gamma} = \frac{\lambda_e}{\lambda_\gamma}$

If a number of groups are emitted then

$$\alpha=\frac{N_k+N_L+N_m}{N_\gamma}=\frac{N_k}{N_\gamma}+\frac{N_L}{N_\gamma}+\cdots=\alpha_k+\alpha_l+\alpha_m \ldots \ldots$$

It can be seen that

1. 'α' is proportional to Z^3. This shows that conversion process dominates in heavy nuclei and γ-emission is favoured in light nuclei.
2. 'α' varies inversely as E*, which means lower energy have more internal conversion activity.
3. 'α' is directly proportional to the multipole order.

4. $0 \rightarrow 0$ transition is not possible because of new spin. But $0 \rightarrow 0$ transition is possible with internal conversion, provided that the electron wave does not vanish at the origin. The life time of $0 \rightarrow 0$ transition is equal to $[\frac{1}{\lambda_k}]$.
5. Comparing the intensity of the γ-ray produced by photo electrons and internal conversion, the process of internal conversion can be experimentally verified.

1.6 Nuclear isomerism:

The time for nuclear transitions by γ-ray emission is usually of the order of 10^{-12} sec. However, some delayed transitions with half-lives as large as several years have been observed. Such excited states are meta-stable states known as nuclear isomers which are nuclides that have the same atomic number and mass number but exist for measurable time before decaying into lower energy states or directly to the ground state. The existence of such isomeric states known as nuclear isomerism.

Ux_1 and Uz have same z values, but different half life time and radiations. Both are given out of UX_1 with the emission of a β^- particle. UX_2 has T=1.18 minutes, emission of three groups of electron of energy of 2.31 MeV, 1.50 MeV and 0.58 MeV. But Uz has T=6.7 hrs, with electrons of energy 0.16 MeV, 0.32 MeV, 0.53 MeV and 1.13 MeV.

Bromine reveals nuclear isomerism. When a target enclosing bromine is bombarded by slow neutrons, it is linked with β-rays of three half-lives such as 18 minutes, 4.9 hours and 34 hours. But chemical analysis shows that each one is the product of bromine. But bromine is expected to have only two isotopes Br^{79} and Br^{81}. The possible β-decays are,

$${}_{35}Br^{79}+{}_{0}n^{1}\rightarrow{}_{35}Br^{80}+\gamma$$

$${}_{35}Br^{81}+{}_{0}n^{1}\rightarrow{}_{35}Br^{82}+\gamma$$

In addition with this one more β-ray with two half lives. Bombardment of bromine with 17MeV γ-rays gives rise to

$${}_{35}Br^{79}+\gamma\rightarrow{}_{35}Br^{79}+{}_{0}n^{1}$$

$${}_{35}Br^{81}+\gamma\rightarrow{}_{35}Br^{80}+{}_{0}n^{1}$$

These products in the above three decays are found to have β-rays with three half lives namely 6.4 minutes, 18 minutes and 4.4 hours. The bromine isomers are chemically separated and it is found that 4.4 hours isomer decays into 18 minutes isomer by emitting two γ-rays in cascade with energies in the order of 0.049 MeV and 0.037 MeV respectively. The 18 minutes isomer which represents the ground state of ${}_{35}Br^{80}$ then decays to ${}_{36}Kr^{80}$ by a β-emissions and then to the ground state of ${}_{14}Se^{80}$ by a positron emission and an orbital electron capture. The decay scheme is given in figure 1.5

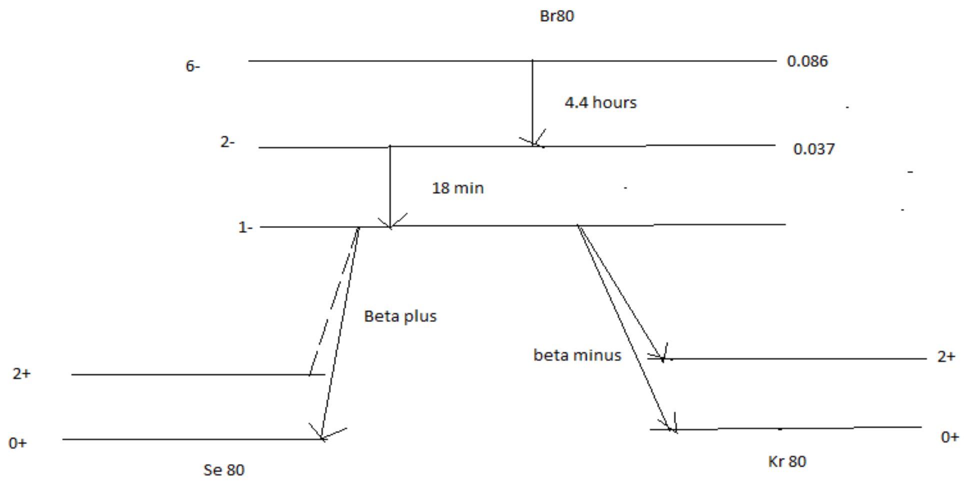


Fig.1.5

Isomeric transitions have large change in the angular momentum, but they have very small change in the energy. Both these factors highly favour internal conversion. Weisacker suggested that an isomer is an atom whose nucleus is in an excited state and has an angular momentum which differs by several units from that of any lower energy level including the ground state. It is also suggested that lesser the spin difference lesser is the half lives light elements are found to have now isomers. Isomeric states have magic numbers 50, 82, 126 and even ‘A’.

1.7 LET US SUM UP:

- Classical mechanics fails to explain the α-decay. Quantum mechanics provide a successful explanation of the problem of α-decay
- α-decay relates the kinetic energy of the α-particle and height of the potential barrier with probability of leakage.
- The periodic fluctuation in the charge and current distribution inside the nuclei set up electric and magnetic field at different points which produce the output electromagnetic wave.
- A nucleus is an excited state can come to the lower energy state by the emission of γ-ray along with some energy to the electron surroundings. So the γ-ray is internally converted into electrons.
- Nuclear species have same atomic and mass number having different radioactive properties are known as nuclear isomerism.

1.8 ANSWER TO CHECK YOUR PROGRESS

1. What is α-decay? Explain

2. How is internal conversion co-efficient of gamma rays obtained? Explain with necessary selection rules.

3. What do you understand by the term nuclear isomerism of gamma rays? Explain in detail.

4. Discuss the Gomow's theory of α-decay.

5. Explain the selection rules of gamma decay.

1.9 Further reading:

1. Physics of the Nucleus – Gupta and Roy, Arunabha Sen Publishers, Kolkata.

2. Nuclear Physics and Application- C.M.Kachhava

UNIT II NUCLEAR DECAY-BETA DECAY

Structure

2.1 Introduction:

An element which is naturally radioactive usually is found to emit either alpha particle or beta particle. In previous chapter, we discussed the alpha particle emission. Here, we discussed about beta particle emission. When a beta particle is emitted by a nuclear atomic number Z, the atomic number of the new atom formed becomes Z+1, but the mass remains practically unaltered since the mass of the beta particle is negligible with comparison with that of a nucleus. Thus, in beta disintegration, the mass number remains the same.

2.2 Objectives:

Fermi's theory of beta decay is discussed- Kurie plots is deliberated- Selection rules is discussed- Electron capture is argued- Parity violation in beta decay is discussed- Neutrinos is discussed- Measurement of neutrino helicity is also discussed.

2.3 Beta decay

The spontaneous decay process in which mass number of nucleus remains unchanged, but the atomic number changes is known as β-decay. The change in the atomic number is accomplished by the emission of an electron (β^-) or the emission of a positron (β^+) or by the capture of an orbital electron. Depending on the above three modes of decay, they are known as β^- decay, β^+ decay and electron capture (k-capture) respectively. In β- decay process, a neutron is converted into a proton, β^- particle and an anti-neutrino.

The decay scheme is, $n \rightarrow p + \beta^- + \bar{\gamma}$

Anti-neutrino is said to be emitted for the conservation of angular momentum.

In β- decay , a proton is converted in to a neutron, a β^+-particle and a neutrino.

The decay scheme is, $p \rightarrow n+\beta^{+}+\gamma$

In electron capture process a proton combines with an orbital electron. The decay scheme is, $p+e^{-} \rightarrow n+\gamma$

Neutrino or anti-neutrino and chargeless particle and has only angular momentum. For neutrinos, spin vectors and angular momentum vectors are oppositely directed whereas in anti-neutrinos, spin and angular momentum vectors are oriented along the same direction.

2.4 Fermi's theory of beta decay:

Fermi's theory is based on the following:

1) A neutron is converted into proton and electron or a proton is converted into a neutron and an electron.

2) The total energy, charge, number of nuclei and the linear momentum are conserved.

3) To conserve angular momentum, a new particle known as neutrino or anti-neutrino is emitted. Neutrino has only angular momentum.

4) The available energy is shared by the neutrino and electron.

5) Electron-neutrino field is very weak.

The transition probability associated with a transmission from the initial state 'i' to the final state 'f' for an electron emitted with a momentum between p and $p+dp_e$ is,

$$\lambda = \frac{2\pi}{\hbar}|H'_{if}|^2\frac{dN_e}{dE_0} \qquad \text{---(1)}$$

where $\frac{dN_e}{dE_0}$ is the no.of quantum mechanical final states per unit interval of total energy.

$|H'_{if}|$ is the matrix element given as

$$|H'_{if}| = \int \psi^*_{FN} H' \psi_{IN} \, d\tau$$

Where ψ_{FN} is the normalized wave function of the final state.

ψ_{IN} is the normalized wave function of the initial state.

Consider the decay scheme,

$$n \rightarrow p + {}_{-1}e^0 + \bar{\gamma}$$

Fermi assumed that the operator is a constant G. Then

$$|H'_{if}| = G \int \psi^*_{FN} \, \psi_{IN} \, d\tau$$

$$|H'_{if}|^2 = G^2 \int |\psi^*_{FN}|^2 |\psi_{IN}|^2 \, d\tau$$

$=G^2 |\psi_e^*|^2 |\psi_\gamma^*|^2 \int |\psi_{FN}^* \psi_{IN} d\tau|^2$

$= G^2M^2 |\psi_e^*|^2 |\psi_\nu^*|^2$

Where, M= $\int \psi_e^* \psi_\gamma d\tau$ is the overlapping integral.

For a given initial state there can be a number of final states.

Assuming box normalization,

$|\psi_e|^2 = |V^{-1/2} e^{iker}|^2 = |V^{-1/2}|^2 = V^{-1}$

$|\psi_e^*|^2 = V^{-1}$

$|\psi_\gamma^*|^2 = V^{-1}$

$|H'_{if}|^2 = \frac{G^2 M^2}{V^2}$

If the distortion of the electron wave function taken into account then, the wave function has to be multiplied by F(Z, E_e) where F(Z, E_e)= $\frac{2\pi\eta}{1-e^{-2\pi\eta}}$ where $\eta = \pm \frac{ze^2mc}{4\pi\epsilon_{e\,h\gamma}}$

The positive sign is for electron and negative sign is for proton. The initial and final state are shown asπ

$|H'_{if}|^2 = \frac{G^2}{V^2}$ F(Z,E) $|M_{if}|^2$

$|M_{if}|$ can be determined from the structure of the nucleus.

Consider a cubical box of length L. The probability of finding to particle inside the box is $|\psi|^2$ where $\psi = A \sin K_x x$ Sin $K_y y$ Sin$K_z z$ where K_x, K_y &K_z are the wave vectors in the x, y and z axis respectively.

Here $K_x=n_x\pi$, $K_y=n_y\pi$, $K_z=n_z\pi$

The square of the momentum,

$p^2= \hbar^2k^2= \hbar^2 [K_x^2+K_y^2 + K_z^2]$

$p^2=\hbar^2\frac{\pi^2}{L^2} [n_x^2 + n_y^2 + n_z^2]$

Each set of $n_x^2 + n_y^2 + n_z^2$ gives the solution for the given momentum state.

The number of quantum mechanical states 'N' having the momentum 'p' is given by the number of unit volumes in the first quadrant of a sphere of radius n_{max}.

N= $\frac{1}{8}.\frac{4}{3}\pi n_{max}^3 = \frac{\pi}{6}(\frac{pL}{\hbar\pi})^3$ $n_{max} = \frac{K_{max}L}{\pi} = \frac{p_{\max L}}{\hbar\pi}$ where $K_xL=n_x\pi$.

N = $\frac{p^3}{6\pi^2\hbar^3}$ where L^3=v. ……………………(2)

The number of final states(dv) with momentum in between p and dp is given by,

$$dN = \frac{3p^2 V.dp}{6\pi^2\hbar^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Let the corresponding energies be $E_0$and E_0+dE_0 The final energy is shared by the electron and neutron, But $E_0=E_e+E_\gamma$

If there is no error in the measurement of electron energy (dEe=0) then,

$$dE_0=dE_\gamma$$

we know that, $E_\gamma^2 = p_\gamma^2 c^2 + m_\gamma^2 c^4$

Differentiating, $2E_\gamma\, dE_\gamma = c^2\, 2p_\gamma.dp_\gamma$

$$dE_\gamma = dE_0 = \frac{p_\gamma c^2}{E_\gamma} dp_\gamma$$

$$\frac{dN_\gamma}{dE_\gamma} = \frac{dN_\gamma}{dE_0} = \frac{V\, p_\gamma^2}{2\pi^2\hbar^3}.\frac{E_\gamma}{c^2}$$

$$\frac{dN_\gamma}{dE_\gamma} = \frac{VE_\gamma^2}{2\pi^2 c^3\hbar^3} = \frac{V(E_0-E_e)^2}{2\pi^2 c^3\hbar^3}$$

$$E_\gamma = p_\gamma.c|$$

$$p_\gamma = \frac{E_\gamma}{c}$$

Equaiton (1) becomes,

$$\lambda = \frac{2\pi}{\hbar}\frac{G^2}{V^2} F(Z,E)\, |M_{ij}|^2 \frac{V(E_0-E_e)^2}{2\pi^2 c^3\hbar^3}$$

$$\lambda = \frac{G^2\, |\mu_{if}|2F(Z,E_e)(E_0-E_e)2}{\pi c^3\hbar^4\, V}$$

The probability for an electron to have a transition to a final energy state is $\lambda_{k,}$ then the probability is given as,

$$\int_0^{p_{max}} p(pe)dpe = \lambda_k dN_e$$

$$= \frac{G^2|M_{ij}|^2\, F(Z,E_e)(E_o-E_e)^2}{\pi c^2\hbar^4 V} = \frac{Vp_e^2 dp_e}{2\pi^2\hbar^4}$$

$$= \frac{G^2|M_{ij}|^2 p_e^2(E_0-E_e)^2 dp_e}{2\pi^3 c^3\hbar^4}$$

$$\frac{p(p_e)}{p_e^2} \propto (E_o - E_\gamma)^2 = p^2\, (E_0 - E_e)^2$$

The momentum spectrum is proportional and $(E_0 - E_e)^2$

2.5 Kurie plots

According to Fermi's theory of β-decay, the probability of emission per time is,

$$P(p_e)\, dp_e = \frac{G^2|M_{ij}|^2}{2\pi^2\hbar^7 c^3} p_e^2 (E_0 - E_e)^2\, F(Z,E_e) dp_e$$

i.e., $\frac{P(p_e)}{p_e^2\, F(Z,E_e)} = (E_0\text{-}E_e)^2$

The plot of $\sqrt{\frac{P(p_e)}{p_e^2\, F(Z,E_e)}}$ with K.E is known as Kurie plot. Kurie plot is a straight line. Let us discuss as Kuries plot for some of the particles.

The Kurie plot for neutron is given below:

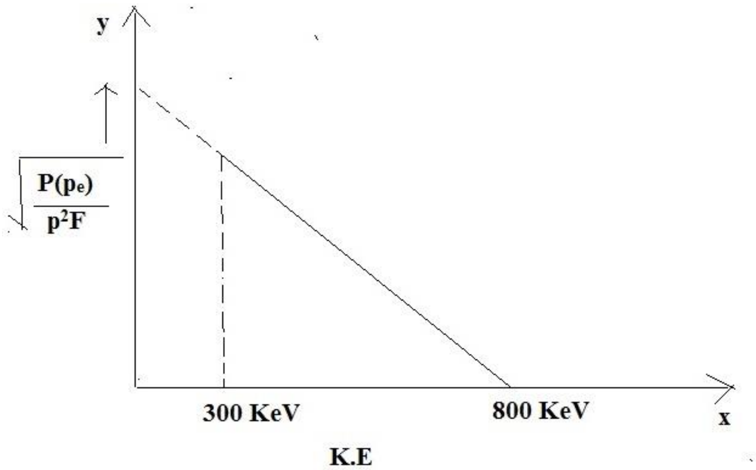


Fig. 2.1

The Kurie plot is a straight line with an end point energy of 800 KeV. Below 300 KeV energy there is deviation from straight line nature. This may be due to the experimental error.

Kurie plot of $_1H^2$

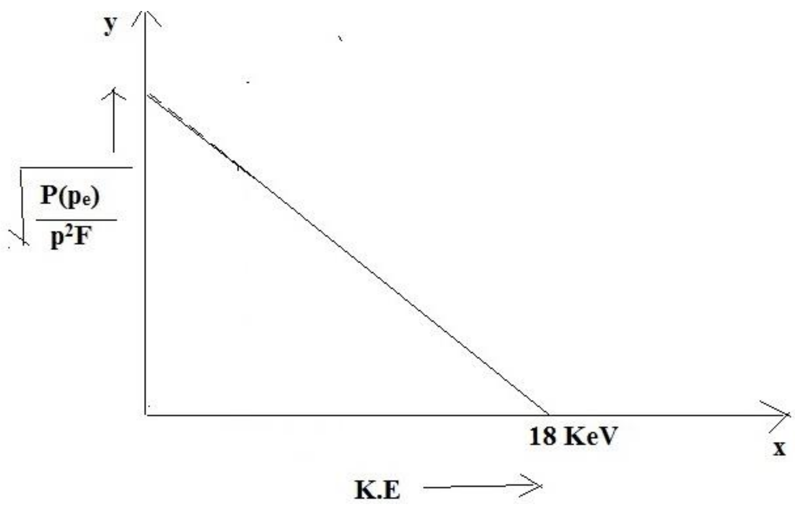


Fig. 2.2

Kurie plot for $_1H^2$ is a straight line having an end point energy of 18 KeV. The straight line nature confirms the validity of the β-decay process.

The Kurie plot of Y^{91} is shown in the figure. It has two end point energies which are 1.19 MeV and 1.549 MeV. Hence two γ-ray emissions are possible with energy 1.549 MeV or 0.35 MeV

Kurie plot of Y^{91}

.

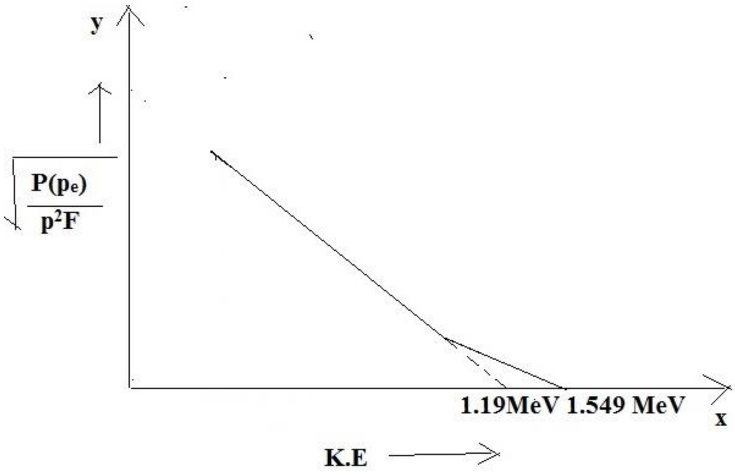


Fig. 2.3

Merits of Kurie Plots.

1. The validity of Fermi's theory of β-decay can be tested.
2. It tells whether the transition is allowed or not.
3. To find the value of $|M_{ij}|^2$ which is used for the determination if the structure of the nucleus.

2.6 Selection rules:

If the spin of the e^- and γ are anti-parallel (single state) then change in the nuclear spin ΔI=0. This is known as Fermi's selection rule.

If the spins of e^- and γ are similarly (triplet state), then ΔI=1, 0, -1 (but $I_1=0$ is not permitted). This refers to Gamow-Teller (GT) selection rule. Experiments show that allowed transitions of the type ΔI=1, obeying G-T selection rule is forbidden in F- selection rule.

$${}_2He^6 \rightarrow {}_3Li^6 + {}_{-1}e^0 + \gamma \quad (0^+ \rightarrow 1^+)$$

ΔI=1 is allowed as per G-T rule, but not allowed as per F-rule ΔI=0 is permitted by F-rule and forbidden by G-T rule.

Similarly, ${}_8O^{16} \rightarrow {}_7N^{14*} + {}_1e^0 + \gamma \quad (0^+ \rightarrow 0^+)$

But, there are transitions which are permitted by both rules.

$n \rightarrow p + \bar{e} + \bar{\gamma} \qquad (1/2^+ \rightarrow 1/2^+)$

${}_1H^1 \rightarrow {}_2He^3 + {}_{-1}e^0 + \bar{\gamma} \quad (1/2^+ \rightarrow 1/2^+)$

${}_{18}S^{35} \rightarrow {}_{17}cl^{35} + {}_{-1}e^0 + \bar{\gamma} \quad (1/2^+ \rightarrow 1/2^+)$

2.7 Electron capture process:

In electron capture process a proton combines with one of the orbital electrons so as to produce a neutron and a neutrino. The decay scheme is,

$$p + e^{-} \rightarrow n + \gamma$$

The initial state has proton and orbital electron whereas final state has neutron and neutrino. The k-electron is the absorbed electron. Hence the wave function of the electron is taken as the k-shell electron wave function.

i.e. $\Psi e = \Psi k = \frac{1}{\sqrt{\pi}}\left(\frac{ZM_0\, e^2}{\hbar^2 4\pi\varepsilon_0}\right)^{1/2}$

The neutrino wave function is $\psi_{\Upsilon} = e^{ikor}.v^{-1/2}$

$$|\psi_{\gamma}| = v^{-1/2}$$

The decay constant associated with the transition from the initial state to the final state is,

$$\lambda_k = 2.\frac{2\pi}{\hbar}|H'_{if}|^2\frac{dN_{\gamma}}{dE_0} \text{ --------------------(1)}$$

Where the matrix element H' = $\int \psi_n^* H' \psi_i \ dT,\ \frac{dN_{\gamma}}{dE_o}$ is the number of final neutrino states per unit interval of energy. The factor 2 appears because there are two electrons in the k-shell.

The operator H' is assumed as a constant and denoted as G.

$$|H'_{if}|^2 = G^2 \int |\Psi_{FN}^*|^2 |\psi_v^*|^2 \, |\psi_{FN}|^2 \, |\psi_k|^2 d\Omega$$

$$= G^2 |\psi_v^*|^2 |\psi_k|^2 |M_{ij}|^2 \text{ where } |M_{ij}| = \int \Psi_{FN}^* \, \psi_{FN} \, d\Omega$$

It can be shown that the number of neutrino states with energy E_{γ} is equal to the number of unit volumes present in the first quadrant of a sphere of radius η_{max}.

i.e., $N_{\gamma} = \frac{1}{8}\,\frac{4\pi}{3}\,(\eta_{max})^3$

$$= \frac{1}{8}\left(\frac{4\pi}{3}\right)\frac{p_{\gamma}^3 L^3}{\pi^3 \hbar^3}$$

$$= \frac{1}{6}\frac{P_{\gamma}^3 V}{\pi^2 \hbar^3}$$

$$dN_{\gamma} = \frac{VP_{\gamma}^2 \, dp_{\gamma}}{2\pi^2 \hbar^3}$$

Substituting the various values in equation (1), we have

$$\lambda_k = 2\frac{2\pi}{\hbar} G^2 V^{-1} \frac{1}{\pi}\frac{z^3 m^3 e^6}{64\pi^3 \varepsilon_0^3 \hbar^6}|\mu_{if}|^2 \frac{1}{2}\frac{p_\gamma^2 dp_\gamma}{\pi^2 \hbar^3 dE_\gamma}$$

Where $dE_0 = dE_\gamma$

$$\lambda_k = \frac{G^2 z^3 m^3 e^6 P_\gamma^2 |M_{ij}|^2}{32\pi^5 \varepsilon_0^3 \hbar^{10} c}$$

This is the probability of electron capture per decay per second.

2.8 Parity violation:

Parity is one of the fundamental properties of the nucleus. Parity is also said to be conserved in a nuclear reaction. But in weak transitions, parity seems to be violated. Parity violation is associated with helicity. In β-decay process, parity is said to be violated, which is explained by C.S.Wu. Wu's experimental arrangement is shown in fig.2.4. Let us imagine a mirror to be placed in front of the real experiment. The real experiment consists of an aligned Co^{60} nucleus emitting β-particle. The sample is cooled to 0.01 K by the process of adiabatic demagnetization. Let us assume that there is anisotropy in the intensity of the emitted β-radiation. Let there be more electrons emitted in the upward direction. i.e, in the direction opposite to that of the angular momentum vector. A magnetic field is applied as shown. Two counters one along the polar direction and another one along the equatorial direction are placed. In the real experiment more electrons are emitted in a direction opposite to that of the angular momentum vector (J).

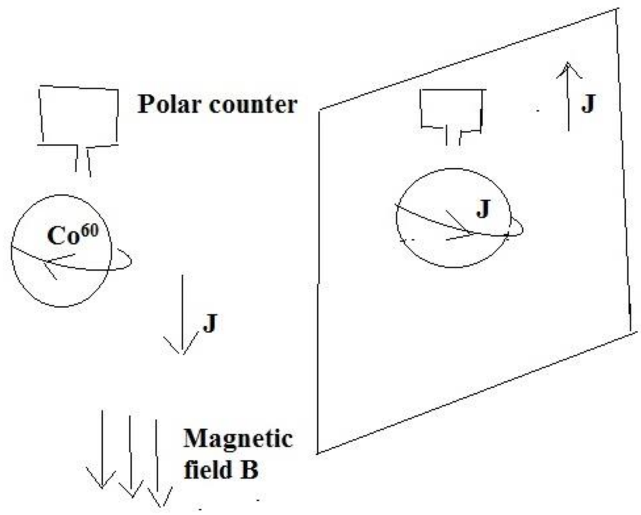


Fig. 2.4

But in the mirror image, more electrons are emitted along the direction of the angular momentum vector (J). This shows the parity violation. The direction of the magnetic field is reversed and the counting rate is conserved. It is found that the counting rate is more for the downward magnetic field than for the upward magnetic field. This indicates the anisotropy of β-emission which confirms the parity violation in β-decay process. The plot of counting rate with time corresponding to the position of the counter and direction of the magnetic field are shown in Fig 2.5 and 2.6 respectively.

Instead of the β-emission if γ-ray emission is considered, the counting rate remains same for upward and downward fields. The parity violation is due to the helicity of the neutrino or anti-neutrino.

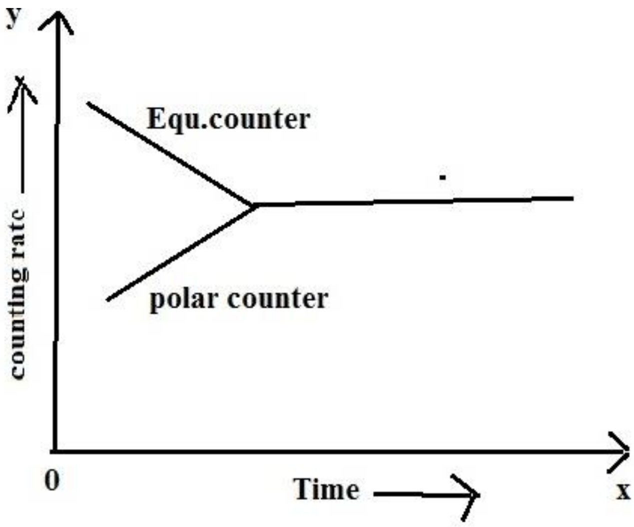


Fig. 2.5

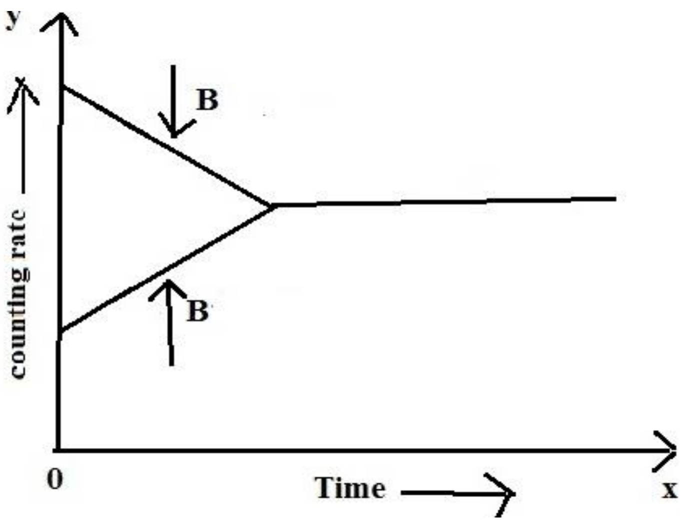


Fig. 2.6

2.9 Neutrino:

The conservation of angular momentum in the β-decay process demands the production of neutrino or anti-neutrinos. Neutrinos have the following properties.

1. chargeless
2. very small mass
3. spin ½
4. small magnetic moment, much smaller than that of the electron.

5. neutrinos have negative helicity (-1) and anti-neutrinos have positive helicity (+1)

Theory of neutrinos:

Consider a weak interact given below.

$P+\bar{\gamma} \rightarrow n+\beta^{+}$

The initial state has proton and anti-neutrino. The final state has neutron and positron. Let λ be the decay constant associated with the decay process.

The decay constant is

$$\lambda= \frac{2\pi}{\hbar} |H'_{if}|^2 \frac{dN_e}{dE_e} \quad \text{---------------------(1)}$$

Where $H'_{if}$the matrix element, dNe is is the number of final states having an energy in between E_e and E_e+dE_e

The matrix element is $|H'_{if}|= \psi_f^* H' \psi_i \, d\tau$ where ψ_f and ψ_I are the unnormalized wave function corresponding to the final and initial states. The operator H' is assumed as a constant G.

Then $|H'_{ij}|= G \int \psi_{FN}^* \psi_e^* \psi_{IN} \psi_v \, d\tau$ where ψ_e is the electron wave function, ψ_v is the neutrino wave function, ψ_{FN} and ψ_{IN} are the normalized wave function of the final and intial states.

$$|H'_{if}|^2=G^2 \int |\psi_{FN}^*|^2 \, |\psi_e^*|^2 |\psi_{IN}|^2 |\psi_v|^2 d\tau$$

$$|H'_{if}|^2=G^2 \, |\psi_e^*|^2 \, |\psi_v|^2 |M_{if}|^2 \quad \text{--------------------(2)}$$

Where $|M_{ij}|= \int \psi_{FN}^* \psi_{IN} d\tau$

Using box normalization,

$$|\psi_e^*|^2 =|v^{-1/2} e^{ik_e r}|^2 = v^{-1/2}$$

$$|H'_{if}|^2= \frac{G^2 |M_{ij}|^2 |\psi_\gamma|^2}{V} F(Z, E_e) \text{------------ (3)}$$

Where F(Z,Ee) is the electron distortion wave function.

The number of final state Ne is given by the number of unit volumes present in the first quadrant of a sphere of radius in

$\eta_{max}(\frac{L}{\pi} \ k_{max}) = \frac{Lp}{\pi\hbar}$

$$N_e= \frac{1}{8} \, \frac{4}{3} \, \pi\eta_{max}^3= \frac{1}{6} \, \pi \frac{L^3 p_e^3}{\pi^3 \hbar^3}$$

$$N_e= \frac{V p_e^3}{6\pi^2 \hbar^3} \quad \text{----------------------(4)}$$

Let dN_e be the number of electron states having energy E_e and E_e+dE_e.

Differentiating equation (4)

$$dN_e=\frac{Vp_e^2dp_e}{2\pi^2\hbar^3} \qquad \text{-------------------(5)}$$

Energy and momentum is related as,

$$E_e^2 = m_e^2c^4 + p_e^2c^2$$

Diffentiating,

$$2E_e\, dE_e=2P_ec^2dp_e$$

$$dE_e =\frac{p_ec^2}{E_e}dp_e \quad \text{-------------------(6)}$$

Substituting equations (3), (5) and (6) in equation (1) we get,

$$\lambda= \frac{G^2|M_{ij}|^2|\psi_\gamma|^2p_eE_eF(Z,E_e)}{\pi\hbar^4c^2} \quad \text{----------------(7)}$$

Let $|M_{ij}|^2$=1 and $F(Z,E_e)$=1

Equation(7) becomes,

$$\lambda= \frac{G^2|\psi_\gamma|^2p_{eE_e}}{\pi^2\hbar^4c^2} \quad \text{----------------------------(8)}$$

if σ is the cross section associated with the reaction and φ is the neutrino flux, then

$$\lambda=\sigma\phi$$

$$= \sigma|\psi_\gamma|^2c \quad \text{---------------------------(9)}$$

Comparing equation (8) and (9) we have

$$\sigma|\psi|^2 c= \frac{G^2|\psi_\gamma|^2p_{eE_e}}{\pi^2\hbar^4c^2}$$

$$\sigma= \frac{G^2p_{eE_e}}{\pi^2\hbar^4c^2} \quad \text{------------------(10)}$$

Equation (10) represents the cross section associated with the reaction. Substituting the appropriate values σ is found to be $0.42 \times 10^{-48} m^2$. This shown that the probability for the reaction to take place is very small. Emission of neutrino can be established only by some indirect methods. The disappearance of the k-electron in the X-ray spectrum given some proof for the existence of neutrino.

2.10 measurement of neutrino helicity:

A system is said to possess helicity if there is parity violation. If the real experiment and its real image are not matching then the system possesses helicity. In β-decay, parity is said to be violated due to the helicity of the neutrino. The neutrino can be measured by perceptive the direction of motion of the neutrino and from its spin direction. The direction of the neutrino can not be directly determined. It must be determined by some indirect methods. The direction of motion of the neutrino is taken as a direction opposite to the motion of the emitted γ-ray. Gold haber measured the helicity using Europium ${}_{63}Eu^{152}$ produced by the bombardment of 10 mgs of EK_2O_3 in a Brook heaven reactor. By the process of electron capture, Eu^{152} is converted into Samarium

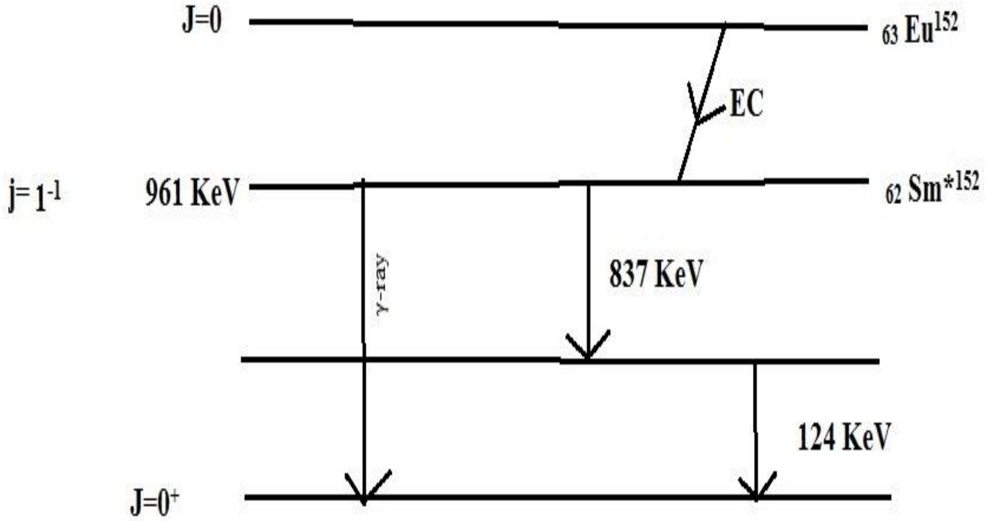


Fig.2.7

The decay scheme is,

$${}_{63}Eu^{152} + {}_{-1}e^{0} \rightarrow {}_{62}Sm^{152} + \gamma$$

Sm^{152} is in the excited state. It comes to the ground state by the emission of a γ-ray of energy 961 KeV or by the emission of two γ-rays of energy 837 KeV and and 124 KeV. The principle of conservation of angular momentum requires that the spin of the excited Sm^{152} must be opposite to that of neutrino. Since they are moving in the opposite directions, the excited nuclear must have the same helicity as that of the neutrino. In the transitions of the 0^+ ground state, γ-ray takes the angular momentum with it. Since the spin of the γ-ray is equal to that of Sm^{152}, then the spin of the γ-ray is opposite to that of neutrino.

The direction of the γ-ray photon was determined by scattering them from a magnetized iron and allowing them to be reabsorbed by Sm^{152}. The 961 KeV γ-rays have a slightly less energy (6.5 eV) then the excitation energy of I^- state because of the recoil loss in the source. If the loss of energy is compensated, then resonance absorption can take place. This is possible only if the γ-ray and the neutrino have the same energy. The experimental arrangement is shown in Fig.2.8. The Europium source is mounted inside an analyzing magnet. The neutrino is emitted in a direction opposite to that of the γ-ray. The emitted γ-ray is scattered by a scatterer. The pulse height of the γ-ray scattered into a scintillation spectrometer is shown in Fig.2.9. The dashed curve is for a scatterer other than Sm^{152} with Sm^{152} scatterer, two resonant peaks are observed at 837 KeV and 961 KeV. The resonant peak indicated the upward emission of neutrino. The helicity of the neutrino is -1.

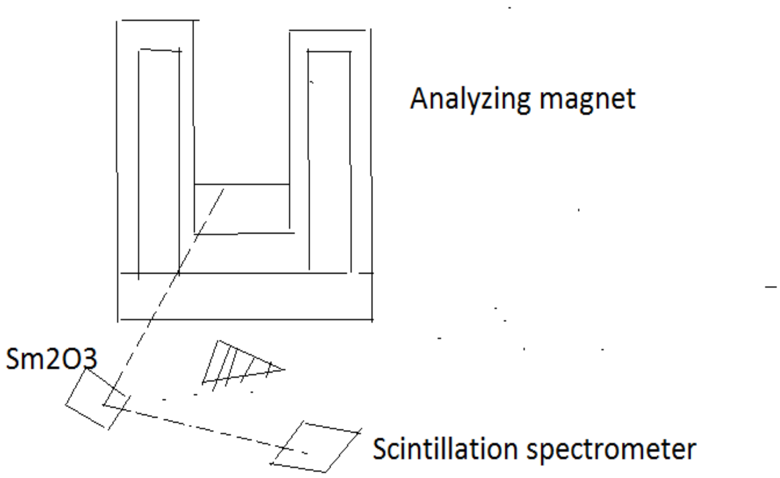


Fig.2.8

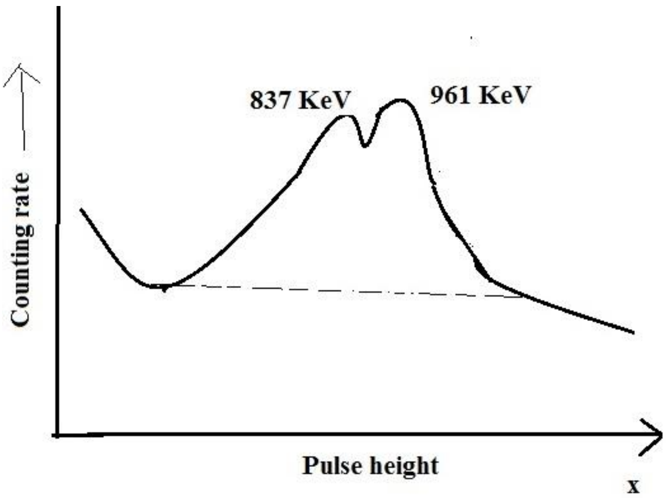


Fig. 2.9

For the measurement of spin direction of the neutrino, let us consider the sequences shown in Fig.2.10. In Fig 2.10 a, the electron with its spin up is captured by the nucleus. In Fig.2.10.b , the neutrino is leaving the nucleus in the upward direction with its spin down. The nucleus is left with an angular momentum J=1, as per the decay scheme. In fig.2.10.c, the transitions to 0^+ takes place, the γ-ray takes this angular momentum and is emitted downwards with negative helicity. The helicity of the γ-rays can be obtained by introducing a magnetized iron absorber in the path of the γ-rays. The absorption co-efficient is large for right handed polarization that for the left handed polarization, when the γ-ray is moving towards the direction of the field. With the reversal of the field, highest counting rate is observed with the field down than it up field. This indicates that γ-rays have left handed polarization. Hence neutrino possesses negative helicity while antineutrino has positive helicity.

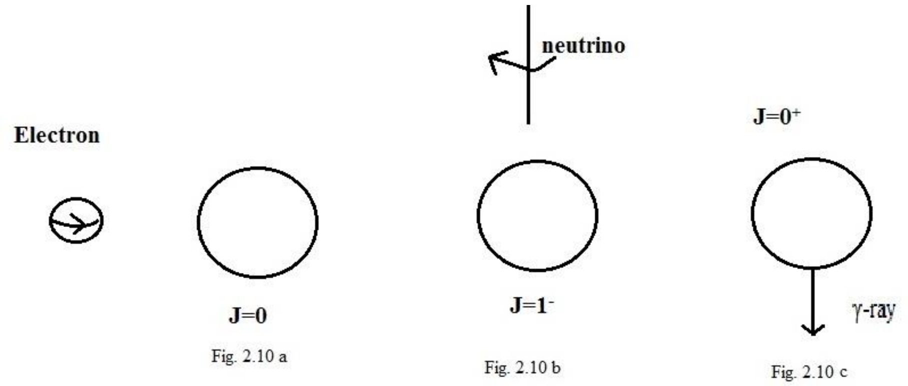


Fig. 2.10

2.11 LET US SUM UP

- ❖ In a beta decay process, the change of atomic number is accompanied by emission of an electron or emission of positron or by the capture of an orbital electron.
- ❖ Kurie plot tells that the validity of the Fermi's theory of beta decay, whether the transitions is allowed or not and used to determine the structure of the nuclei.
- ❖ The existence of neutrino and anti-neutrino are discussed. Owing to conserve the angular momentum of beta decay process, the neutrino and antineutrino are generated.
- ❖ Parity is said to be conserved in a nuclear reaction. But in weak interaction, parity is said to be violated, and is also associated with neutrino helicity.

2.12 Review questions:

1. Give Fermi's theory of β-decay.
2. How far Fermi's theory has been verified experimentally?

3. Explain the measurement of neutrino helicity.
4. Explain the various β-decay process.
5. Give an account on parity violation in β-decay.
6. Explain the selection rules of β-decay.
7. Explain the process of electron capture with relevant theory.
8. What are neutrinos? Discuss the theory of neutrinos.

2.13 Further readings:

1. Nuclear physics D.C.Tayal, Himalya house, Bombay

2. Nuclear physics V.Devanathan, Narosa Publishers, New Delhi

UNIT –III NUCLEAR LIQUID DROP AND COLLECTIVE MODELS

Structure:

3.1 Introduction:

The behavior of a very heavy, excited compound nucleus can be understand by the regarding the nucleus in its cross features to be like a liquid drop. Each nucleon in the interior of a nucleus. Liquid drop and shell models of the nucleus are, in their very different way, able to account the nuclear behavior. The collective model combine features of both models in consistent scheme that has proved quite successful.

3.2 Objectives:

Liquid drop model is discussed- Bohr wheeler theory is deliberated- Schmitt lines are argued- Magnetic dipole moment is discussed- Electric quadrupole moment is discussed- Collective model is discussed.

3.3 Liquid drop model:

There is some similarity between the Liquid drop and Nucleus structure. i.e.,

1. The nucleus is supposed to spherical in shape in the stable state, just as liquid drop is spherical due to symmetric surface tension forces.
2. The force on the surface tension acts on the surface of the liquid drop. Similarly, there is potential barrier at the surface of the nucleus.
3. The density of the water molecule and Nucleus are independent of its volume.
4. The intermolecular force in liquid and Nuclear force are short range.

5. Molecules evaporate from a liquid drops on raising the temperature of the liquid due to the increased energy by thermal agitation. Similarly, when energy is given to nucleus by bombarding it with nuclear projectile, a compound nucleus is formed which emits nuclear radiation almost immediately.
6. When a small drop of liquid is allowed to oscillate it breaks up into smaller drops of equal size. The process of nuclear fission is similar the nucleus breaks up into smaller nuclei.

3.4 Bohr wheeler theory:

The liquid drop model of the nucleus is proposed by N.Bohr and Kalcker. This model provides many phenomena which cannot be explained by other models. Bohr suggested that all quantities of a given liquid have the same density under the same conditions; the properties of the nucleus could be compared with a liquid drop. The following are the similarities between liquid drop and atomic nucleus.

1. Nuclear forces are analogous to the surface tensional force of liquid.
2. The behavior of a nucleon is similar to that of molecules in liquids.
3. Density is independent of the atomic number(A) as density of liquid is independent of size and shape of a liquid drop.
4. Binding energy per nucleon is analogous to the latent heat of vaporization.
5. Disintegration of a nucleus is analogous to the evaporation of molecule from the surface of a liquid.
6. Energy of the nucleus corresponds to the thermal energy of a drop.
7. Formation of compared nucleus corresponds to the condensation of drops.

Let a spherical drop of radius R_0 be deformed. If R is the distance of the deformed surface from the centre at angle $(\theta,\emptyset)$. Then the difference in the distance is,

$$q(\theta,\emptyset) = R(\theta,\emptyset)-R_0= \sum_{l=0}^{\infty}\sum_{m=-l}^{l} q_{l,m}Y_{l,m}(\theta,\emptyset)$$

For simplicity, let us assume a cylindrically symmetric deformation for which m=0 and the harmonic oscillation of $q_{l,0}$ is given as,

$$q_{l,0} = q_l \cos(\omega_l t)$$

Let ω be the characteristic frequency. The restoring force is supplied by the surface tension which opposes the deformation of the surfaces.

The surface energy is $E_s = \alpha.S = \alpha.\ 4\pi R^2$ ----------------------(1)

The characteristic frequency ω is given by Rayleigh as,

$$\omega = [\frac{4\pi\alpha}{3\mu}\ l(l-1)(l+2)]^{½} \qquad \text{-------------------------------(2)}$$

where μ is the reduced mass of the drop= MA where M is the mass of a molecule.

From Weizsacker-Bethal mass formula, the surface energy is related to the surface tension coefficient as,

$$Es= a_2 A^{2/3} \qquad \text{-------------------------(3)}$$

Where a_2=13MeV

From eq.(1) and eq.(3), we have

$$E_s= 4\pi R^2\alpha = a_2\ A^{2/3}$$

$$a= \frac{a_2 A^{2/3}}{4\pi R^2} \qquad \text{-------------\textbackslash-----------(4)}$$

But $R=R_0A^{2/3}$, where R_0 is the unit nuclear radius=1.3 ×10^{-15}m

Equation (4) is written as,

$$\alpha= \frac{a_2 A^{2/3}}{4\pi Ro^2 A^{2/3}} = \frac{a_2}{4\pi Ro^2}$$

Substituting for value of α in equation (2), we ger

$$\omega=[\frac{4\pi}{3MA}\frac{a_2}{4\pi Ro^2}\ l\ (l-1)\ (l+2)]^{1/2}$$

$$\omega= [l(l-1)\ (l+2)\ \frac{a_2}{3Ro^2MA}]^{1/2}$$

The corresponding energy= $\frac{h}{2\pi}\omega$

$$= \frac{h}{2\pi}[\ l(l-1)\ (l+2)\ \frac{a_2}{3Ro^2MA}]^{1/2}$$

$$\text{Energy}= 14.7\ [\frac{l(l-1)(l+2)a_2}{A}]^{½}\ \text{MeV}$$

This model gives the correct value of atomic mass and binding energy. This predicts α and β emissions. This explains nuclear fission. This model is not successful in determining the actual excited states.

3.5 Magnetic moment:

Most of the odd nuclei possess angular momentum and magnetic moment in their ground state. The total angular momentum is the vector sum of the orbital angular momentum and spin angular momentum. Magnetic moment can be established in the presence of an excited magnetic field.

The total magnetic moment,

$$\vec{\mu}_j = \vec{\mu}_l + \vec{\mu}_s$$

$$= g_l \vec{L} + g_s \vec{S}$$

Where $\vec{L}$ and $\vec{S}$ are the vectors which have no components in the field direction, but they have components in the field direction.

The component of the magnetic moment along the direction of J-vector is,

$$g\sqrt{j(j+1)} = gl\sqrt{l(l+1)}\ \cos(l,j) + gs\ \sqrt{s(s+1)}\ \cos(s,j)$$

Applying the law of cosines and rearranging

$$\text{Cos }(l,j) = \frac{J(j+1)+l(l+1)-s(s+1)}{2\sqrt{j(j+1)l(l+1)}}$$

Similarly, $$\text{Cos }(s,j) = \frac{j(j+1)+s(s+1)-l(l+1)}{2\sqrt{j(j+1)s(s+1)}}$$

$$g = g_l\left[\frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)}\right] + g_s\left[\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}\right]$$

Magnetic moment along with J-vector is,

$$\mu_j = g\vec{J} = g\sqrt{j(j+1)}$$

If J is the total angular momentum of the nucleus, then replacing j by J we have,

$$\mu_j = g\sqrt{J(J+1)}$$

Maximum value along the direction of the J-vector

$$\mu_j = g\sqrt{J(J+1)}\ \frac{J}{\sqrt{J(J+1)}}$$

$$\mu_j = gJ$$

$$= J\left[\frac{J(J+1)l(l+1)-s(s+1)}{2(J+1)}\right] g_l + J\left[\frac{J(J+1)+s(S+1)-l(l+1)}{2(J+1)}\right] g_s \quad \text{---------(1)}$$

where g_l and g_s are the Lande's splitting factors.

Stretch case:

Let $j = l + \frac{1}{2}$

then $l = j - \frac{1}{2}$

Substituting for l in equ (1) we get

$$\mu_{str} = \left[(J+1) - \frac{1}{2}\right] g_l + \frac{g_s}{2} \qquad \text{--------------------- (2)}$$

For proton g_l=1 and g_s=5.586

For neutron g_l=0 and g_s=-3.826

Jack knife case:

When j= l- $\frac{1}{2}$

l= (j+$\frac{1}{2}$)

substituting for l in equ.(1), we get

g_{Jack}= $\frac{J}{(J+1)}$ [(J+$\frac{3}{2}$) g_l-$\frac{g_s}{2}$] -------------------(3)

Equations (2) and (3) show that the magnetic moment is a function of J.

3.6 Schmidt lines:

The plot of μ_j with J is shown in the fig.3.1, the lines are known as Schmidt lines. The agreement between the experimental and theoretical values may be due to the error in the measurement of magnetic moment or due to the assumption that the nucleons move in a spherical symmetric potential which is not true.

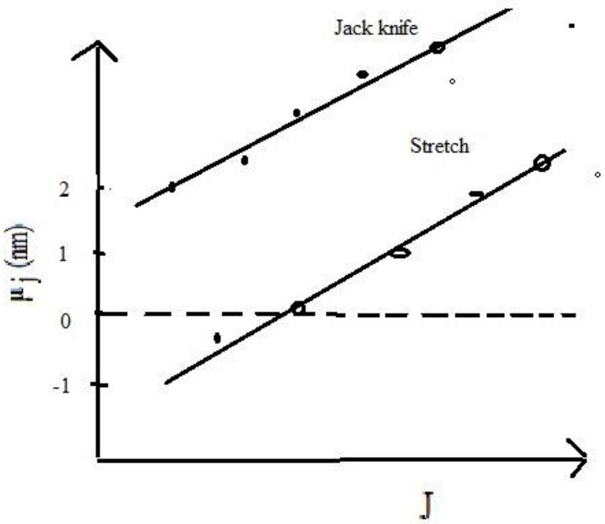


Fig.3.1

3.7 Electric quadrupole moment:

The deviation from spherical symmetry is expressed by a quantity called the quadrupole moment.

Q=$\frac{1}{e}\int(3z^2 - r^2)\rho d\tau$ ---------------(1)

Q- measure of eccentricity

ρ- charge density of nuclei.

Q=0 spherically symmetric charge distribution. A charge distribution stretched in the z-direction (prolate) will give the positive quadrupole moment and oblate give the negative quadrupole moment.

According to independent particle model, the nuclei with closed shells should have spherical charge distribution and hence zero quadrupole moment. Experiments also confirm this. The quadrupole moment of an odd A, odd Z nuclide is due to the last unpaired proton. The measured value corresponds to m_j=J. If $\bar{L}$ and $\bar{S}$ are parallel (j=l+$\frac{1}{2}$), m_l=1 and m_s=1/2.

The proton wave function becomes,

$$\Psi = \frac{V(r)}{r} N_l p_l^l(\cos\theta)e^{i\phi.\alpha} \quad \text{-----------------------------(2)}$$

Where α is the spin wave function and N_l is the normalization constant given as,

$$\int |N_l p_l^l(\cos\phi)e^{i\phi.\alpha}|^2 \, d\Omega=1$$

The quadrupole moment is given as,

$$Q= \int |(3Z^2 - r^2)|\psi|^2 \,|d\tau \quad \text{----------------------------------(3)}$$

$$=[\int_0^\infty r^2| V(r)|^2 \, d\tau] \, [\int N_l^2 \, (3cos^2 \theta\text{-}1)p_l^l \, (\cos\theta)^2 \, d\Omega]$$

But $(2l+1) \cos \theta \, p_l^l (\cos\theta) = p_{l+1}^l (\cos\theta)$

and $\int p_n^m (\cos\theta)^2 \, d\Omega = \frac{4\pi(n+m)}{(2n+1)(n-m)}$

$$N_l^2 = \frac{(2l+1)}{[4\pi(2l)]}$$

Simplifying equ(3) becomes

$$Q=\frac{2l}{(2l+3)} \bar{r}^2 = \frac{2j-1}{2j+2} \bar{r}^2$$

Where j=l+1/2

and $\bar{r}^2 = \int_0^\infty r^2 |U(r)|^2 \, dr$

If $\bar{L}$ and $\bar{J}$ are antiparallel then

$$Q= \frac{(2j-1)}{(2j+1} \bar{r}^2 = \frac{(2l-2)}{2l+1} \bar{r}^2$$

Where Z=2, 8, 20, 50, 82, Q=0. When a new shell begins to start, Q becomes negative. As Z increases Q becomes positive and increase until it reaches maximum and after that it becomes negative.

3.8 Collective Model:

The success of the liquid-drop and nuclear-shell model seems to lead to a serious dilemma. The liquid-drop model can account for the behavior of the nucleus as a whole, as in nuclear reactions and nuclear fission. In the latter certain nuclei actually divide into two smaller nuclei and the division can be described in terms of the deformation of a drops, an explanation in terms of the motion of a single nucleon, or any number of nucleons, seems impossible. The liquid-drop model is said to describe the collective behavior of the nucleus, and the excitation of the nucleus is treated as surface oscillations, elastic vibrations, and other such “collective” modes of motion. On the other hand, many nuclear phenomena seem to show that nucleons behave as individual, and nearly independent, particles. Hence there are two entirely different ways of regarding nuclei, with a basic contradiction between them. The reasonable conclusion is that the two different models or pictures are incomplete parts of a larger or more general model, which includes both the liquid-drop and independent particle aspects; this new model is called the collective model. It is assumed, in this model, that the particles within the nucleus exert a centrifugal pressure on the surface of the nucleus as a result of which the nucleus may be deformed into a permanently non-spherical shape; the surface may undergo oscillations (Liquid-drop aspect). The particles within the nucleus then move in a non-spherical potential like that assumed in order to account for the quadrupole moments. Thus, the nuclear distortion reacts on the particles and modifies somewhat the independent-particle aspect. The nucleus is regarded as a shell structure capable of performing oscillations in shape and size. The result is that the collective model can be made to describe such drop-like properties as nuclear fission, while at the same time preserving the shell model characteristics and, in fact, improving on the earlier shell model.

The simplest type of collective motion which has been identified experimentally is connected with rotations of deformed nuclei. If the rotational collective motion is sufficiently slow, it will not affect the internal structure of the nucleus. According to classical physics, the rotational energy is proportional to the square of the angular velocity ω, so that

$$E_{rot} = \frac{1}{2} I\omega^2 \qquad \text{----------------------(1)}$$

The parameter I denotes an-effective moment of inertia of the nucleus which can be calculated if a specific model is assumed for the internal structure. Rotational energy levels are obtained when the angular momentum is quantized and the result is, for even-even nuclei

$$E_{rot} = \frac{\hbar^2}{2I} J(J+1) \quad \text{--------------------------(2)}$$

Where J is the total angular momentum quantum number of the nucleus. In the case of a spheroidal nucleus, the deformation is symmetric with respect to reflection in the nuclear centre. As a result, J is restricted to even values 0, 2, 4, 6,…… and the parity should be even. According to the theory, the deformation (deviation from spherical shape) should be greatest and the rotational levels most easily observed for nuclei with numbers of nucleons far from closed shells. The first excited state should be a 2^+ state; the second excited state should be a 4^+ state with energy 5.4/3.2 =3.1/2 times that of the first excited state. The third and fourth states should have $I=6^+$ and 8^+ respectively, and energies 7 and 12 predictions, as is evident from Fig.3.2 which shows the experimentally observed energy ratios for excited rotational states in even-even nuclei with mass numbers in the regions 150< A<190 and A>220. These are just the regions farthest from closed shells. The theory also predicts correctly the properties of the low-lying levels of nearly spherical nuclei close to closed shells. Another successful application is to the problem of the cross section for Coulomb excitation and for the probability of E2 transitions. Finally, the theory has been applied successfully to problem of magnetic moments, quadrupole moments, and isomeric transitions.

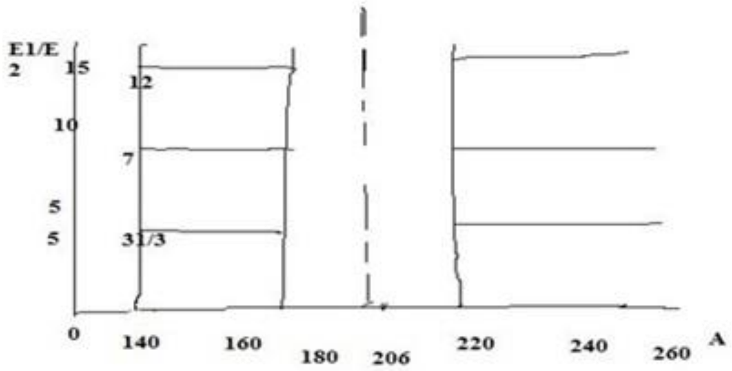


Fig.3.2

3.9 Summary:

- ❖ The behavior of a very heavy, excited compound nucleus can be understood by regarding the nucleus in its group features to be like a liquid drop.
- ❖ The successes of the liquid-drop and nuclear shell model seems to lead to a serious dilemma.
- ❖ The nucleus possesses angular momentum due to spin and also possesses a magnetic moment.
- ❖ The deviation from spherical symmetry is expressed by a quantity called the electric quadrupole moment.

3.10 Review questions:

1. Give the theory of liquid-drop model of a nucleus and discuss it.
2. What are Schmidt lines? Explain

3. Obtain an expression for the magnetic moment of neutron.
4. Give the theory of nuclear quadrupole moment.
5. How is liquid drop and a nucleus compared? Explain

   .

3.11 Further reading:

1. Nuclear Physics and Applications - C.M.Kachhava
2. Nuclear physics – V.Devanantha, Narosa Publishers, New Delhi.

UNIT IV NUCLEAR SHELL MODEL

Structure

4.1 Introduction:

A basic goal of nuclear physics is to determine the motion of the nucleons in a nucleus, and from the motion to derive nuclear properties, both in the ground and excited states. The problem is more complex than in the atomic case, due to the lack of dominant central force and existence of two classes of particles- neutrons and protons. Each class separately obeying the exclusion principle.

4.2 Objectives:

Shell model is discussed- Single particle model, validity and limitations are deliberated- Rotational spectra is discussed- Magic numbers is discussed- spin-orbit coupling is deliberated-Angular momentum of nucleus ground states is discussed- Magnetic moments of the shell model is discussed.

4.3 Shell Model:

For certain numbers of neutrons or protons, known as magic numbers, the nuclei are found to be most stable. Nuclei with Z or A equal to any one of these magic numbers such as 2, 8, 20, 28, 50, 82, 126, 184, they are found to be more stable. The property cannot be explained by the liquid drop model of the nucleus whereas it can be explained on the basis of the shell model.

The idea of the shell model depends on some nuclear properties such as discontinuities in the nuclear mass, nuclear isomerism, the spins and parities of the ground state, magnetic and quadrupole moments of the nuclei and the existence of magic numbers.

4.3.1 Salient features of shell model:

The shell model of the nucleus is an attempt to account for the existence of magic numbers, and some other nuclear properties of nucleon behavior in a common force field. This model assumes that each nucleon stays in a well-defined quantum state. But, unlike the atom, the nucleus has no obvious massive central body acting as field force centre of charge.

In the shell model, therefore, each nucleon is considered as a single particle that moves independently of others in the time-averaged of the remaining (A-1) nucleons acting as a core, and is confined to its own orbit completing several revolutions before being disturbed by others by way of collisions. This implies that the mean free path before collisions of nucleons is much larger than the nuclear diameter. It amounts to assuming the interaction among the nucleons to be weak. This sound paradoxical as nuclear matter is super-dense (-10^{17} Kg/m^3) and experiments indicate that a nucleus is virtually opaque to any incident nucleon. This weak interaction paradox was saved by invoking Pauli's principle by Weisskopf. He argued that nucleons are fermions and by exclusion principle, no two neutrons and protons can stay in identical quantum state. Hence experimentally expected strong interaction among nucleons in a nucleus cannot show itself since all the quantum states (low lying) into which the scattered nucleon after collision may go are already occupied.

4.4 Extreme single particle model:

In the shell model, the nucleons of an atom are placed in shells in which the nucleons with opposite spins pair off. The valence nucleon in the unclosed shells in the average potential of the nucleons moving independently. Therefore the spins and magnetic moment of the even-even nuclei are zero.

The potential used in the Schrodinger wave equation may be a square well potential of infinite or finite width.

Inside the nucleus, nucleons are arranged in different shells. Each shell has a sub-shell. The number of nucleons in each shell is (2j+1) where j is the angular momentum and some of the nucleons are arranged in the sub-shell.

1. For Even-even nuclei, the angular momentum $=0^+$
2. For odd-even nuclei (with odd Z or odd N), angular momentum is due to the last unpaired nucleon.
   Parity is $= (-1)^l$
3. For odd-odd nuclei,
   The angular momentum =vector sum of orbital and spin motion
   $J= (j_1-j_2)$ if $(j_1+j_2+l_1+l_2)$ is even
   $J= (j_1+j_2)$ if $(j_1+j_2+l_1+l_2)$ is odd

Parity is $(-1)^{ln+lp}$

The nucleons are arranged according to their energy levels in different shells. The low spin shells fill faster than the high spin shells. The order is given as,

$$({}^{1}s_{1/2})^{2}\ ({}^{1}p_{3/2})^{4}\ ({}^{1}p_{1/2})^{2}\ ({}^{1}d_{5/2})^{6}\ ({}^{2}s_{1/2})^{2}\ ({}^{1}d_{3/2})^{4}\ ({}^{1}f_{7/2})^{8}\ ({}^{2}p_{3/2})^{3}\ ({}^{1}p_{3/2})^{2}$$

There may be some exceptions.

4.5 Validity and limitation of single particle model:

Validity:

1. It explains very well the existence of magic numbers and the stability and high binding energy on the basis of closed shells.
2. The single particle model provides explanation for the ground state spins and magnetic moments of the nuclei. The neutron and protons with opposite spins pair off so that the mechanical and magnetic moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.
3. Nuclear isomerism, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39-49, 69-81, 111-125 has been explained with shell model by the large difference in nuclear spins o isomeric states as their A-values are close to the magic numbers.

Limitations:

1. The model does not predict the correct value of spin quantum numbers I in certain nuclei e.g. ${}^{23}Na_{11}$ where predicted values is I=5/2. The correct value is ½.
2. The following four stable nuclei ${}^{2}H_{1}$, ${}^{6}Li_{3}$, ${}^{10}B_{5}$ and ${}^{14}N_{7}$ do not fit into this model.
3. The model cannot explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation. It also fails to explain the observed large quadrupole moment of odd- A nuclei, in particular of those having A-values far away from the magic numbers.
4. If all inter-nucleon couplings are ignored, the model is called single particle shell model. If however, couplings are considered, it is known as independent particle shell model.

4.6 Rotational spectra:

In the rare-earth and actinide regions, the nuclei exhibit large electric quadrupole moments indicating permanent deformation in shape. The low-lying energy levels of even-even nuclei correspond to rotational spectrum with spin-parity 0^+, 2^+, 4^+, 6^+….. If I is the moment of inertia of the rotating object, then its kinetic energy is given by

$$E=1/2\ I\ \omega^2 = ½\ I(\frac{l^2}{I})^2 \quad \text{since } l= I\omega$$

$$= \frac{\hbar^2}{2I}\ l\ (l+1) \quad \text{------------------------------}(1)$$

Equation (1) is obtained by replacing the square of the orbital angular momentum operator l^2 by its expectation value l (l+1) $\hbar^2$. Once again replacing l by j, we obtain the quantum mechanical energy separation of a right rotator.

$$E_j= \frac{\hbar^2}{2I}\ j(j+1) \text{ ----------------------------}(2)$$

The ground state of even-even nuclei is always 0+ and the mirror symmetry of the nucleus restricts the sequence of rotational states to even values of j.

4.7 Magic numbers:

The inert gases, with 2, 10,18,36,54...., electrons, having closed shells show high chemical stability, nuclei with 2, 8, 20, 50,82 and 126 electrons- the so called magic numbers- of the same kind (either proton or neutrons) are particularly stable. The binding energy is found to be unusually high implying high stability which is reflected in high abundance of isotopes with these proton numbers and isotones with these neutron numbers. Nuclei both with Z and N= each a magic number, are said to be doubly magic.

4.8 Spin-orbit coupling:

A nucleon inside the nucleus has orbital motion and spin motion. An interaction is existing between the orbital motion and the spin motion. This interaction can be represented can be represented by a potential. The interaction energy is W=-μ_s.B= -f(r) $\bar{s}\,.\,\bar{L}$ where $\bar{s}$ and $\bar{L}$ are the spin and orbital momentum vectors and f(r) is the radial potential function. The potential which describes the single particle model is V(r) =f(r) S.I where V(r) and f(r) depend on the radial distance and size of the particles. The total angular momentum is,

J = l$\pm$s where j=(l+s) denotes stretch case

= l-s represents the jackknife case

Then $\quad S.I= \frac{1}{2}(j^2 - l^2 - s^2)$

$$= \frac{1}{2}\ [j(j+1)-l(l+1)-s(s+1)]$$

$= \frac{1}{2}$ l for j=l+1/2

$= -\frac{1}{2}$ (l+1) for j=l-1/2

The effect of the spin-orbit coupling is to split the energy levels.

4.9 Angular momentum of nuclear ground states:

The resultant angular momentum of a nucleus is called (for historical reasons) the nuclear spin. It is designated by I. Both protons and neutrons, like electrons, have spin ½. In addition, protons and neutrons possess orbital angular momentum associated with their motion of the nucleus. The resultant nuclear angular momentum (or spin) is obtained by combining, in a proper way, the orbital angular moments and the spins of the nucleons composing the nucleus. The nuclear spin is designated by a quantum number l such than the magnitude of the nuclear spin is $\hbar$ $[l(l+1)]^{1/2}$. The component of the nuclear spin in a given direction is given by ml$\hbar$ where ml= $\pm$l, $\pm$ (l-1), ……..$\pm l/2$ or 0, depending on whether l is a half-integer or an integers. Therefore there are 2l+1 possible orientations of the nuclear spin. As explained earlier these same rules are valid for all angular momenta in quantum mechanics. Experimentally the values of l are integers (If A is even) or half integers (if A is odd) ranging from zero as in ^{4}He and ^{12}C up to 7 as in ^{178}Lu. Even-odd nuclei (i.e., nuclei that have an odd number of either protons or neutrons) all have half-integer angular momenta, and it is reasonable to assume that the nuclear spin coincides with the angular momentum of the last or unpaired nucleon, a result which seems to hold in many cases. Odd-odd nuclei have two unpaired nucleons (one neutron and one proton) and the experimental results are a little more difficult to predict, but their angular momenta are integers, as one would expect, since there is an even total number of particles. Any theory of nuclear forces, to be satisfactory, must account for the experimental values of l.

4.10 Magnetic moments of the shell model:

Unless the nuclear spin is zero, we expect nuclei to have magnetic dipole moments, since both the proton and the neutron have intrinsic magnetic moments, and the proton is electrically charged, so it can produce a magnetic moment when it has orbital motion. The shell model can make predictions about these moments. Using a notation similar to that used in atomic physics, we can write the nuclear magnetic moment as

$$\mu = g_j\, j\, \mu_N$$

Where μ_N is the nuclear magneton that was used in the discussion of hadron magnetic moments. g_j is the Lande g-factor and j is the nuclear spin quantum number. For brevity we can write simply $\mu = g_j\, j$ nuclear magnetons.

We will find that the shell model does not give very accurate predictions for magnetic moments, even for the even-odd nuclei where there is only a single unpaired nucleon in the ground state. We will therefore not consider at all the much more problematic case of the odd-odd nuclei having an unpaired proton and on unpaired neutron.

For the even-odd nuclei, we would expect all the paired nucleons to contribute zero net magnetic moment, for the same reason that they do not contribute to the nuclear spin. Predicting the nuclear magnetic moment is then a matter of finding the correct way to combine the orbital intrinsic components of magnetic moment zero net magnetic moment, for the same reason that they do not contribute to the nuclear spin. Predicting the nuclear magnetic moment is then a matter of finding the correct way to combine the orbital and intrinsic components of magnetic moment of the single unpaired nucleon. We need to combine the spin component of the moment, $g_s s$, with the orbital component, $g_l\, l$ (where g_s and g_l are the g-factors for spin and orbital angular momentum) to give the total moment g_j. The general formula for doing this is

$$gj= \frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)}\, gl + \frac{j(j+1)-l(l+1)+s(s+1)}{2j(j+1)} gs \text{ --------------------------(1)}$$

which simplifies considerably because we always have $j= l\pm\frac{1}{2}$. Thus

$$jg_l= g_l l+g_s/2 \quad \text{for } j=l+1/2 \text{ -------------------------} \qquad (2)$$

$$jg_j= g_l j\,\left(1+\frac{1}{2l+1}\right) -g_s j\left(\frac{1}{2l+1}\right) \quad \text{for } j=l-1/2 \text{ --------------} \quad (3)$$

Since g_l=1 for a proton and 0 for a neutron, and g_s is approximately +5.6 g for the proton and -3.8 for a neutron, Equation (3) yield the results is the g-factor for nuclei with an odd proton(neutron)

$$Jg \text{ (proton)}= l+\frac{1}{2}\times 5.6= j+2.3 \text{ for } j=l+1/2$$

$$Jg \text{ (proton)}= j\left(1+\frac{1}{2l+1}\right)- 5.6 \times j \left(\frac{1}{2l+1}\right) = j- \frac{2.3\, j}{j+1} \text{ for } j=l-1/2$$

4.11 LET US SUM UP

- The shell model is able to account for several nuclear phenomenon in addition to magic numbers.
- The agreement between the theoretical and experimental is found to be poor in Schmidt lines which reveal that the assumption the nucleons move in spherical symmetric potential which is not true.
- The spin- orbit potential which is to split the energy levels.

4.12 Review questions:

1. What are magic numbers?

2. Explain how spin-orbit coupling can be accounted on the basis of shell model.
3. Discuss the features of the collective nuclear model. How does represent rotational nuclear spectra?
4. Give an account on the single particle model of the nucleus.
5. Discuss the detailed note on shell model.

4.13 Further reading:

1. Nuclear and particle physics An introduction- Brian R Martin
2. Physics of the Nucleus –Gupta and Roy, Arunabha Publishers, Kolkata.

BLOCK II NUCLEAR FISSION, FUSION

UNIT V Nuclear reaction and mechanism

Structure:

5.1 Introduction:

New opportunities occur when we go elsewhere natural radioactive decays and study nuclear reactions by bombarding nuclei with other nuclear particles such as proton, neutron and α-particle, etc., A most important neutron induced nuclear reaction is fission. When two light nuclei fuse to form a larger nucleus, energy is released. The enormous amount of energy is released during fission reaction than fusion reaction.

5.2 Objectives: Nuclear fission and Fusion are discussed- Nuclear reaction is deliberated- reaction mechanisms is discussed- Compound nuclei and direct reactions are discussed- Simple theory of deuteron is discussed- Tensor forces (qualitative) is discussed.

5.3 Nuclear fission and Fusion:

The phenomenon of the division or disintegration of a heavy nucleus into two nucleus of comparable masses is termed nuclear fission or simply fission, in analogy with cell division in biology.

The schematic equation for the fission process may be written as

$^{235}U_{92}+{}^{1}n_{0}\rightarrow ({}^{236}U_{92})^{*} \rightarrow X+Y+\text{neutron}$

Where $^{1}n_{0}$ is a slow neutron, $^{236}U_{92}$ a highly unstable isotope of uranium and X, Y are the fission fragments.

A fusion reaction is the one in which two lighter nuclei fuse together to form a heavy nucleus. Since colliding nuclei are positively charged and thus have a break through the electrostatic repulsion between them to fuse together, the kinetic energy of the colliding particles must be high enough to overcome this repulsion. A large kinetic energy means high temperature. It is seen that the initial kinetic energy increases rapidly with atomic number and so the most promising candidates to undergo fusion are lighter nuclei like hydrogen and helium.

The fusion process is expected to release a large amount of energy because a part of the mass of the fusing nuclei is converted into energy. Fission is started by neutrons and heat or kinetic energy of the neutrons plays no part in carrying out the fission chain reaction, it only depends upon the greater number of neutrons released. The fusion reaction on the other hand is analogous to combustion and can take place only at temperatures of the order of hundreds of millions of degree centigrade. Since fusion reaction requires a very high temperature, they are also termed at thermo-nuclear reactions.

5.4 Nuclear reaction:

Our knowledge of nuclear structure is mostly the outcome of experiments of bombarding a nucleus with various projectiles such as protons, neutrons, deuterons, α-particles etc., After the bombardment, it may so happen that the mass number and/or atomic number of the target nuclei changes, that is, nuclear transformation takes place. We then say a nuclear reaction has occurred.

A nuclear reaction is thus a process that takes place when a nuclear particle such as proton, neutron, deuteron, α-particle, a nucleus etc., comes in close contact (within 10^{-15} m) with another, and energy and momentum exchanges occur. The final products of the reaction are also some nuclear particle or particles that leave the point of contact in different directions. The process results in the transmutation of the target nucleus.

The changes that occur in a nuclear reaction usually involves strong nuclear force. Processes that involve weak interactions (e.g β-decay) or are purely electromagnetic (coulomb scattering) are not usually included under nuclear reactions. Changes of nuclear states due to electromagnetic interaction however are included.

A general equation representing a nuclear reaction is of the form:

$$X+x \rightarrow Y+y$$

$$\text{Or, } X(x,y)\,Y$$

where X is the target nucleus, x is bombarding particle, Y the residual product nucleus and y the ejected particle.

In a nuclear reaction, charge number, mass number, total energy etc, are conserved.

Rutherford observed for the first time the transmutation of elements in 1919 by α-ray scattering on nitrogen.

$$\alpha + {}^{14}N \rightarrow {}^{17}O + p$$

With the construction of particle accelerators, it has become possible to observe a variety of nuclear reactions. All these reactions obey certain conservation laws: conservation of energy, momentum, angular momentum, parity, charge and baryon number. The Q-value of nuclear reaction is

$$a + A \rightarrow b + B,$$

is defined by

$$Q = m_a + m_A - m_b - m_B$$

$$= T_b + T_B - T_a - T_A$$

Where m denotes the mass and T denotes the kinetic energy of the particles, If $Q > 0$, the reaction is said to be exothermic reaction and if $Q < 0$, it is said to be endothermic reaction. In first case, the nuclear mass is converted into kinetic energy whereas in the second case, the kinetic energy is converted into mass as per the reaction of mass and energy $E=mc^2$.

5.5 Reaction mechanism:

Several models for reaction mechanism have been formulated to study the nuclear reactions at different energies. We shall consider briefly three such models-compound nucleus model, optical model and direction reaction model..

If the incident nucleon is of 1 MeV, its debroglie wavelength is about 4 fm which is comparable to the nuclear radius and hence does not see the individual nucleons in the nucleus. It interacts with the entire nucleus through the formation of compound nucleus mechanism has three interesting features.

1. The cross section depends upon the compound nucleus that is formed and not the way, it is formed. Two difference entrance channels forming the same compound nucleus will yield the same cross section and the energy dependence.
2. The cross section depends sensitively on the energy and shows sharp peaks known as resonances.
3. The angular distribution of emitted particle will be isotropic.

For a slightly large energy of the incident nucleon, the nucleus can be replaced by an optical potential with real and imaginary parts. One can use a partial wave analysis for the scattering of the incident nucleon. The real part of the optical potential yields the elastic section and imaginary part gives the absorption cross section which accounts for the sum of the cross sections for all other inelastic and reaction channels.

For a 20 MeV incident nucleon, the de Broglie wavelength is about 1 fm and the incident nucleon begins to 'see' the individual nucleons in the target nucleus and the interaction time is of the order of 10^{-22} sec. An incident neutron can pick up a protons from the target nucleus and emerge as a deuteron in the exit channel. This is known as a pick up reaction. If the deuteron is used as a projectile, it can sometimes loose a neutron to the target nucleus and emerge as a proton in the exit channel. This is known as stripping reaction. The pick-up and stripping reaction are examples of direct reaction. The direct reaction, the angular distribution reaction are the examples of direct reaction. In direct reaction, the angular distribution is sharply peaked. This is in contrast tool the isotropic distribution that the compound nucleus mechanism yields.

In the case of heavy nucleus, a new phenomenon of nuclear forces is observed by the absorption of low energy thermal neutrons. We shall consider the mass and charge distribution in such nuclear fission.

Nuclear reactions are very useful in investigating the nuclear structure, the nuclear energy levels and their spin and parity. Instead of neutrons, protons, deuterons and alpha particles as projectile, heavier nuclei are used for some investigations and this study is known as heavy ion reactions which are used for some investigations and this study is known as heavy ion reactions.

In addition, electrons and muons are used as electromagnetic probes of nuclear targets. In the medium energy range, they provide valuable information on the charge distribution and momentum distribution of nucleons in the nucleus. At very high energies in the GeV range, they provide ample evidence for the composite structure of nucleus that they are made up of patrons (point particles) known as quarks and gluons.

5.6 Compound nucleus and direct reaction:

When an incident particle approaches a target nucleus, it goes absorbed by the target nucleus. This forms a compound nucleus which is in the excited state. This is known as formation process. The compound nucleus which is in the excited state decays with the emission of a particle. This is known as decay process. The mode of disintegration of the compound nucleus depends only on the specific way in which it has been formed. Since the decay process is independent of the formation process, the reaction cross section is given as,

$$\sigma_{(p,q)} = \sigma_c (E_c, p)\, G_e(E_e, q) \qquad \text{-----------------------}(1)$$

Where p and q are the angular momentum states of the incident and emitted particles respectively. E_c is excitation energy of the compound nucleus, σ_c and G_c are the probability for (cross section) the formation process and decay process respectively. In order to calculate the cross section associated with the formation process, the following assumptions are made.

1. The nucleus has a well-defined surface with a radius R.
2. Inside the nucleus, the potential is negative i.e., $v = -v_0$
   Since the energy of the incident particle is shared by all the nucleus in the target, the probability of a particle re-emitted without loss of energy becomes negligible. If the outgoing particle has full energy, then the process is known as elastic scattering and if the particle does not at all enter into the nuclear potential, it is known as potential scattering.

For l=0 neutrons, the Schrodinger wave function for a particle inside the nucleus is,

$$\frac{d^2u_0}{dr^2}+\frac{2m}{\hbar^2}(E+V_0)u_0=0$$

$$\frac{d^2u_0}{dr^2}+\alpha^2u_0=0 \quad \text{---}(2)$$

where α is the wave vector inside the nucleus, given as, $\alpha^2=\frac{2m}{\hbar^2}(E+V_0)$.

Assuming that the compound elastic scattering is negligible, within the surface of the nucleus, the incoming wave is represented by the solution of equ.(2) as,

$$u_0= e^{-i\omega t}$$

Let us define logarithmic derivative f_0 as

$$f_0 = \frac{r}{u_0}\left(\frac{du_o}{dr}\right)_{r=R}$$

$$= \frac{R}{e^{-i\omega}}(-i\alpha)\, e^{-i\omega R}$$

$$f_0= -i\alpha R$$

Bur $\eta_0 = \left(\frac{f_{o}+ikR}{f_o-ikR}\right) e^{-2\alpha R}$

Substituting for f_0, we get

$$\eta_o= \left(\frac{-i\alpha R+ikR}{-i\alpha R-ikR}\right) e^{-2\alpha R}$$

Then $|\eta_o|= \frac{\alpha-k}{\alpha+k}$ -------------------------------(3)

The reaction cross sections for l=0 neutron is defined as,

$\sigma_{r,0}=\frac{\pi}{k^2}[1-|\eta_0|^2]$ ---------------------------------(4)

Substituting for $|\eta_0|$ in equ.(4) and rearranging,

$\sigma_{r,0}=\frac{4\pi\alpha}{k(\alpha+k)^2}$ ---------------------------------(5)

If the energy of the incident particle approaches zero, $\hbar^2k^2$ approaches zero and hence $\sigma_{r,0}=\infty$. Hence this theory is invalid at very small energies. If the incident beam consists of neutrons of different angular moment states then the total cross section becomes,

$\sigma_c\,(Ec,p)=\sum_{l=0}^{\infty}\sigma_{nJ}$ --------------------(6)

The above equation represents the cross section associated with the formation process.

The nucleus can decay in many numbers of final channels. Let us assume that nucleus be breaking up through a channel q with an energy Eq and a decay constant $\lambda_{q.}$

By Heisenberg's uncertainty principle

$\Delta E.\Delta\varepsilon=\hbar$

$\Delta E\tau=\hbar$

$\Gamma(\frac{1}{\lambda})=\hbar$

$\Gamma=\hbar\lambda$ where $\Gamma=\Delta E$ and $\tau=\Delta\epsilon$

Hence $\Gamma_q=\hbar\lambda q=\hbar/\tau_q$ =level width. Considering all the possible channels

The total level width is $\Gamma=\sum_q\Gamma_q$

The probability of decay is $G_e(E_e,q)=\Gamma_q/\Gamma$

According to reciprocity theorem

$\frac{k_e^2\sigma_c(p)}{\Gamma_p}=\frac{k_e^2\sigma_c(q)}{\Gamma_q}=\ldots\ldots.\ U(E_c)$ -----------------------(7)

Where U(Ec) is the function of the excitation energy of the nucleus and this does not depend on the channels. Then,

$G_e(E_c,q)=\frac{\Gamma_q}{\sum_\gamma\lambda_\gamma}$

$=\frac{\lambda_q}{\sum_\gamma\lambda_\gamma}$

$G_e(E_e,q)=\frac{k_q^2\sigma_c(q)}{\sum_\gamma k_\gamma^2\sigma_c(\gamma)}$

Hence σ_c(p,q) can be obtained.

5.7 Simple theory of deuteron (ground state)

Deuteron consists of a proton and a neutron. The deuteron does not exist in the bound state. The properties of deuterons are

1. charge is +ve
2. mass=2.014735 a.m.u
3. spin=1 (in the units of $\hbar$)
4. statistics =(Bose-Einstein)
5. Radius=2.1 Fermi
6. binding energy=2.225$\pm$ 0.003 MeV
7. magnetic dipole moment= 0.87736 nuclear magneton
8. electric Quadrupole moment Q=0.0028 barn
9. parity=even

A theoretical description of deuteron is given here. Deuteron is a two body problem. The following assumptions are made.

1. Since the mass of the proton and neutron are approximately equal, the reduced mass is (m/2) where m is the average mass of a nucleon.

2. The force between the nucleons is a short range, attractive force, acts along the line joining the two particles and does not depend on the orientation i.e., it is a central force. This force can be derived from the potential.

The Schrodinger wave equation for a two body problem is given as

$$-\frac{\hbar^2}{2m}\nabla^2\Psi+V\Psi = E\Psi \text{ -----------------------(1)}$$

Where m is the reduced mass. V is the potential describing the force between the two particles and E is the total energy of the system. For the ground state deuteron E= E_B=-2.225MeV. The potential V depends on 'r' as well as on some other parameters. For simplicity let us assume that the potential V is the function of 'r'.

For the ground state (l=0 states) deuteron let us consider a square well potential. V(r) for r < c and V(r) for c < r < (b+c) where c is the core radius, 'b' is the width of the potential and V_0 is the depth of the potential. The plot of the potential with distance of separation (r) between the particles inside the deuteron is shown in fig.5.1

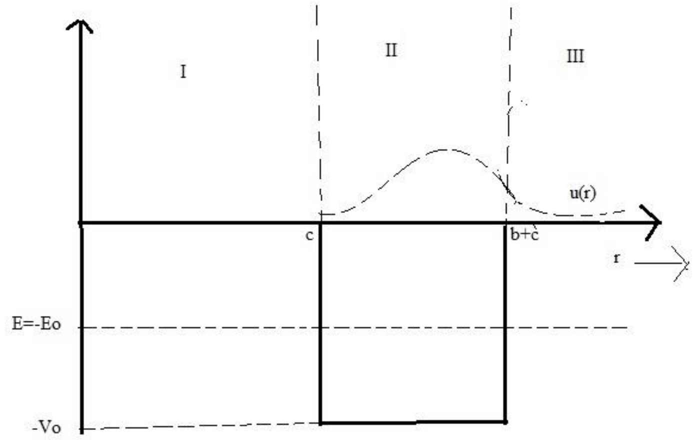


Fig. 5.1

The S.E. for the region II (c < r < (b+c)) is

$\frac{d^2uII}{dr^2}+\frac{2m}{\hbar^2}(V_0\text{-}E_B)\, u_{II}=0$ --------------------------------------(2)

The solution can be obtained using the boundary condition

At r= c, $u_{II}=0$ Hence the solution becomes

u_{II}= A sink(r-c) -------------------------------------(3)

Where $k^2=\frac{2m\,(Vo-E_B)}{\hbar^2}$ and A is a constant

For the region III r > (b+c)

The S.E is

$\frac{d^2uIII}{dr^2}+\frac{2m}{\hbar^2}E_B)\, u_{III}=0$

The solution is $u_{III}= Be^{-\alpha r}+B'\, e^{\alpha r}$ ---------------------(4)

Where B and B' are constants and $\alpha^2=\frac{2mE_B}{\hbar^2}$ with the boundary condition that at r =α, u_{III} = 0, equation (4) becomes $u_{III} = Be^{-\alpha r}$ ----------------(5)

At r =c+b the boundary condition we get

A sink (b+c-c) = B e-α(b+c)

A sin kb= B e-α (c+b) ---------------------------------(6)

Since the wave function is continuous at r=b+c

We have $\left(\frac{du_{II}}{dr}\right)_{r=b+c}=\left(\frac{du_{III}}{dr}\right)_{r=b+c}$

The above condition gives

Ak cosk (c+b-c) =-α B $e^{-\alpha(c+b)}$

Ak coskc= -αB$e^{-\alpha(c+b)}$ -----------------------------------(7)

Equ(7)/Equ (6) gives

k cot kb= -α ----------------------------------(8)

or cot kb= $-\frac{\alpha}{k}=\sqrt{\frac{2mE_B}{2m(V_0-E_B)}}=\sqrt{\frac{E_B}{(V_0-E_B)}}$

Equ (8) relates the binding energy with the width of the potential and depth of the potential.

If V_0= 40 MeV, then b=1.895.

5.8 Tensor forces:

Experimental evidence in favour of Non-central forces: In a structure made up of two particles, one expects that the total magnetic moments to be the vector sum of the magnetic moments due to spin and magnetic moments due to orbital motion of charged particles. Since deuteron is supposed in l=0 states, no contributions from orbital motion is expected, and we should have magnetic moment of deuteron equal to $\mu_0+\mu_\tau$=-0.8797 μ_B. but the experimentally measured value is μ_d= 0.8574 μ_N. The small difference in expected and measured magnetic moments suggests that there is some orbital motion in ground state of deuteron and out inference that, I=0 in the deuteron ground state is not completely correct. Kellong in 1939 discovered serious discrepancies between the predicted and observed fine structure in radio frequency magnetic resonance spectrum of deuterium. The discrepancies in the fine structure could be explained by assuming that deuteron posseses a quadrupole moment, Q=2.84 ×10^{-27} cm^2. If the ground state of deuteron is assumed in l=0 state, the system will be spherically symmetric and quadrupole moment will be zero. The non-zero value of quadrupole moment in the ground state deuteron can be accounted only when there exists a 4 % probability that deuteron will be found in D-state while 96 % it is in S-state. It is noted that in both the states (l=0 or l=2), spin of the deuteron is 1.

The foregoing explanation based on mixing of l values violates the principle of conservation of orbital angular momentum. It is known that angular momentum is changed only by some torque acting on the system, which is defined as

$$\tau = r\times F - rF_\theta$$

$$=\frac{\partial V}{\partial \theta}$$

A changing orbital angular momentum implies that the potential V is a function of θ and not merely a function of r. Since a central forces is defined as one for which V is a function only of r. F_θ is a non- central force. It is called a tensor force.

The angle θ is measured from the direction of spin vector S and force is function of Sr. The fig(5.2) shows the two situations for the same separation r. The positive value of quadrupole moment of deuteron indicates that in deuteron indicates that in deuteron they prefer to line up their spins one after the other rather than side by side; implying, thereby, that is former case force is attractive and in the latter it is repulsive.

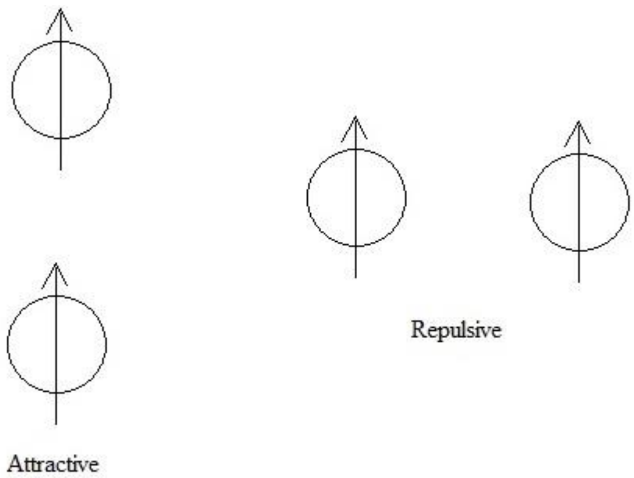


Fig.5.2

Now the assumption that nuclear force is derivable from a potential, requires that potential requires that potential should be such that it remains invariant under the rotation and reflection of the coordinates used to describe it. The combination of space and spin coordinates which satisfy the above requirement is

$$(\bar{\sigma}_1\times\bar{r}).(\bar{\sigma}_2\times\bar{r})= r^3\ \bar{\sigma}_1\bar{\sigma}_2-(\bar{\sigma}1.\bar{\sigma}2)-(\bar{\sigma}._1\bar{r}).(\bar{\sigma}_2.\bar{r})\text{-----------}(2)$$

The non-central potential is taken such that its average over all directions of r vanish. Now

$$\frac{1}{4\pi}\int \bar{\sigma}.1\bar{r}).(\bar{\sigma}2.\bar{r})\,d\Omega= ½\, r^2\ \bar{\sigma}_{1.}\bar{\sigma}_2)$$

The equation (3) makes it possible to define the tensor operator such that

$$\bar{S}_{11}=\frac{3}{r^2}(\bar{\sigma}._1\bar{r}).(\bar{\sigma}_2.\bar{r})\ \bar{\sigma}_1-\bar{\sigma}_2$$

If r_n is the unit vector in the direction of r, then

$$\bar{S}_{12}=\ 3(\bar{\sigma}_1.\bar{r}_u).(\bar{\sigma}_2.\bar{r}_u)-\bar{\sigma}_1\bar{\sigma}_2$$

Thus, non-central force is derivable from the potential.

$$V = V_T(r)\ \bar{S}_{12}$$

As σ is zero for singlet states, there emerges a generalization that tensor force does not exist in singlet state. Physically, it is supported by the fact that in singlet states no preferred directions exists from which θ can be measured.

5.9 LET US SUM UP

- Nuclear structure is mostly the outcome of experiments of bombarding a nucleus with various projectiles such as protons, neutrons, deuterons, α-particles etc.,
- After the bombardment, it may so happen that the mass number and/or atomic number of the target nuclei changes, that is, nuclear transformation takes place. We then say a nuclear reaction has occurred.
- When an incident particle approaches a target nucleus, it goes absorbed by the target nucleus. This forms a compound nucleus which is in the excited state. This is known as formation process.
- Tensor force is non-central force.

5.10 Review questions:

1. Give the simple theory of deuteron
2. What are tensor forces?
3. Write the properties of deuteron.
4. Give the theory of a compound nucleus formation and its decay.
5. What is nuclear fusion? Discuss proton-proton cycle and carbon-nitrogen cycle.
6. Explain the compound nucleus concept.

5.11 Further readings:

1. Physics of the nucleus- Gupta and Roy, Arunabha Sen Publishers, Kolkata.
2. Nuclear physics- V.Devanantham, Narosa Publishers, New Delhi.

UNIT VI NUCLEAR FORCE

Structure:

6.1 Introduction:

The only two forces, we have previously encounted cannot account for the existence of nuclei. The only explanation is to recognize that there is a third force in nature, known as the nuclear force. The force must be strong at distances of the nuclear size. Since it must more than compensate the coulomb repulsion between protons. On the other hand, molecular structure can be accurately accounted for by the electromagnetic force alone. So we may conclude that at distances of the order of the spacing between nuclei in molecules (10^{-10}m) the nuclear force is negligible. It is therefore a short-range force.

6.2 Objectives: Nature of nuclear force is discussed- form of nucleon-nucleon potential is deliberated- Charge independent and charge symmetry of nuclear forces are discussed- Normalization of deuteron wave function is discussed.

6.3 Nature of nuclear force:

The principle of electromagnetic force between the protons is strong coulomb repulsion, which tends to tear the nuclear apart. The gravitational force is an attractive one between every pair of nucleons, but it is an attractive one between every pair of nucleons, but it is smaller by a factor of about 10^{39} than the electrical force between the two protons, its effects are completely negligible in all nuclear and atomic phenomena.

Thus, the only two forces we have previously encounted cannot occur for the existence of nuclei. The only explanation is to recognize that there is a third force in nature, known as the nuclear force. The force is very strong at distances of the order of nuclear size there must be compensate the coulomb repulsion between two protons.

On the other hand, molecular structure can be accurately accounted for by the electromagnetic force alone so we may conclude that a distance of the order of the spacing between nuclei in molecule (10^{-10} m) the nuclear force must be negligible it is a short-range force, falling off more rapidly with the distance than $1/r^2$. Some aspects of nuclear force are still incompletely understood, that several qualitative features can be described.

(i). Short range:

The nucleus would pull in additional protons and neutrons. But within this range it must be stronger than electric force. Otherwise the nucleus can never be stable. Here short range means that nuclear force is appreciable only when the interacting particle can very close, at separation of the order of 10^{-15} m or less at greater distances, the nuclear force is negligible. We may infer that nuclear force is of short range because at distances greater than 10^{-14} m, corresponding to nuclear dimension, the interaction regulating the scattering of nucleon and grouping of atoms into molecule is electromagnetic. If the nuclear force were of long range, the nuclear interaction between the atomic nuclei would be fundamental in discussing molecular formation.

(ii). Relation orientation of the spin of the interacting nucleus:

Scattering experiments and by analysis of the nuclear energy levels. It has been found that the energy of a two nucleon system in which the two nucleons have their spins parallel is different from the energy of such a system in which one has spin up and spin down. In fact n-p system has bound state, the deuteron in which two nucleons have their spins parallel (S=1), but no such bound state seems to exist if the spins are anti-parallel (S=0).

(iii). Nuclear force is not completely central:

It depends on the orientation of the spins relation to the line joining the two nucleons, scientists have concluded this by nothing that even in the simplest nucleus (deuteron), the orbital angular momentum (l) of the two nucleons relative to their center of mass is not constant, Therefore, to explain the properties of ground state of the deuteron. Such as magnetic dipole and electric quadruple moment. We must use a linear combination of s (l=0) and (l=2) wave function. Part of the nuclear force is strongly spin-orbit interaction. Another part tensor force closely resembles the interaction between the two dipoles.

(iv) The nuclear force is repulsive core:

This means that at very short distances, much smaller than range, the nuclear force becomes repulsive. This assumptions has been introduced to explain to constant average separation of nucleus, resulting in a nuclear volume proportional to the total number of nucleons, as well as to account for certain features of n-n scattering.

In spite of , all the information about nuclear force, the correct expression for the potential energy for nuclear interaction between two nucleons is not yet well known. One is the Yukawa potential

$$E_p(r) = -E_0 r_0 \frac{e^{-r/r0}}{r} \text{-------------------(1)}$$

Where E_0 and r_0 are two empirical constants. The constant r_0 is the range of nuclear force and E_0 gives the strength of interaction. The decreasing exponential factor $e^{-r/ro}$ drops. The Yukawa potential energy leads to zero faster than electric potential energy which varies as 1/r.

6.4 Nucleon-nucleon potential:

For simplicity, Let us assume that the nuclear force is derivable from a potential and it is velocity independent. Then the nucleon-nucleon potential should be a function of the three dynamical variables, the relative position coordinate r and the spins σ_1 and σ_2 of the two nucleons, for the moment, the iso-spin. Let us assume the nucleon 1 to be fixed at the coordinate r_1, the wave function for the meson field is given by

$$(\nabla^2-\mu^2)\phi_0 = (r_1)\ \alpha\ (\frac{G}{2M})\ \sigma_1.\nabla_1\ \delta\ (r-r_1) \text{ ----------------------------(1)}$$

The solution of the field equation (1) can be written as

$$\varphi_0 = \frac{G}{8\pi m}(r_1)\ \alpha\ \sigma_1.\nabla_1 \frac{e^{-\mu(r-r_1)}}{|r-r_1|} \text{ ------------------------------(2)}$$

The interaction with nucleon 2 is obtained by inserting 2 in to the pseudoscalar-pseudovector coupling equ

$$V=\frac{G^2}{16\pi M^2}(r_2-r_1)\ (\sigma_2.\nabla_2)\ (\sigma_1.\nabla_{1)}\ (\frac{e^{-\mu|r_2-r_1|}}{|r_2-r_1|} \text{----------------------(3)}$$

Since $\nabla_1 = -\nabla_2 = \nabla$, we have

$$V = \frac{g^2}{(2M)^2}(r_2 - r_1)\ (\sigma_1.\nabla_1)\ (\sigma_2.\nabla_2)\ (\frac{e^{-\mu r}}{r}) \text{ -------------------(4)}$$

Where $r(=r_2-r_1)$ is the distance between the two nucleons and ∇ is to taken with respect to r. The quantity g^2 has been defined in Yukawa potential equ. It can be shown that the effect of successive operators is to yield the tensor term in the nucleon-nucleon potential.

$$(\sigma_2.\nabla_2)\ (\sigma_1.\nabla_1)\ (\frac{e^{-\mu r}}{r}) = \{\ \frac{1}{3}\ \sigma_1.\sigma_2\nabla_2 + S_{12}\ (\frac{1}{r^2}+\frac{\mu}{r}+\frac{\mu^2}{3})\}\ (\frac{e^{-\mu r}}{r}) \text{ ---------(5)}$$

Where S_{12} is the tensor potential defined by

$$S_{12} = \frac{3}{r^2}\ (\sigma_1.r)\ (\sigma_2.r) - \sigma_1.\sigma_2 \text{ -------------------------(6)}$$

Further,

$$\nabla^2\left(\frac{e^{-\mu r}}{r}\right) = \mu^2 \left(\frac{e^{-\mu r}}{r}\right) = -4\pi\delta(r) \text{ ----------------------(7)}$$

Substituting (5) and (7) into (4), we obtain the total nucleon-nucleon potential

$$V(r) = -\frac{4\pi}{3}\frac{g^2}{(2M)^2}(r_1.r_2)\ (\sigma_1.\sigma_2\ )\ f(r); \qquad r < \lambda_N \quad \text{-----------(8)}$$

$$V(r) = \frac{g^2}{(2M)^2}(r_1.r_2)\ \{\frac{1}{3}\mu^2(\sigma_1.\sigma_2) + S_{12}\ (\frac{1}{r^2}+\frac{\mu}{r}+\frac{\mu^2}{3})\}\ (\frac{e^{-\mu r}}{r}); \quad r > \lambda_N \text{ --(9)}$$

Where λ_N is the Compton wavelength of the nucleon (λ_N =0.2 fm). Equation (8) holds good for short inter nucleon distances and it is assumed that the nucleons are not point particles but have a small spread denoted by the form factor f(r). This results in the replacement of δ(r) by f(r) in equ.(8) is repulsive since ($r_1.r_2$) ($\sigma_1.\sigma_2$)=-3. Equation (9) corresponds to inter nucleon distance greater than λ_N and it is of greater interest. The first term in equ (9) is the central force and it is attractive for all states of even L, since they correspond to either iso spin triplet and spin singlet or isospin sglet and spin triplet states for which

$$(r_1.r_2)\ (\sigma_1.\sigma_2) = -3$$

For state of odd L, the central force is repulsive since they correspond to either isospin triplet and spin singlet states for which

$$(r_1.r_2)\ (\sigma_1.\sigma_2) = 1,$$

Or singlet isospin and triplet spin states for which

$$(r_1.r_2)\ (\sigma_1.\sigma_2) = 9.$$

The second term in equ (9) denotes the tensor force. It is remarkable that the exchange of pseudo scalar meson leads to the experimentally observed tensor forc

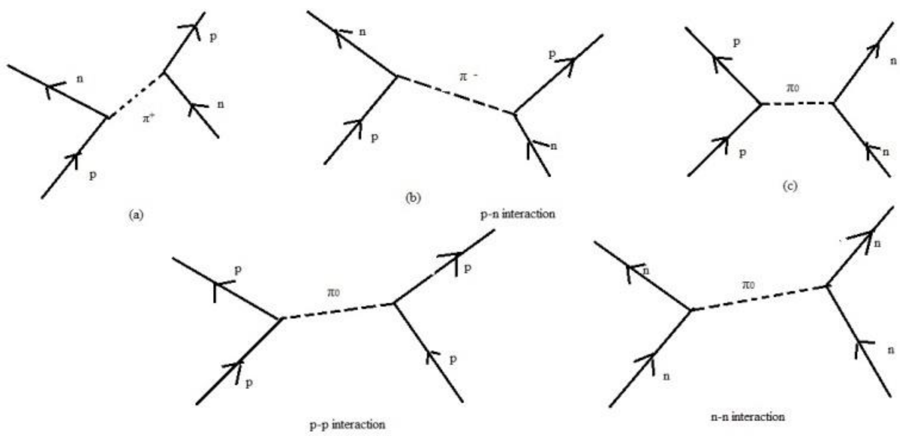


Fig.6.1

Long before the pion was discovered and its pseudo scalar nature established, the properties of nucleon-nucleon potential were used to guess the nature of the Yukawa quantum. What has been discussed hitherto is the symmetric theory involving the exchange of charged and neutral pions and the above diagrams

(Fig.6.1) the both the p-p and n-n interactions arise only from an exchange of neutral pions whereas n-p interaction involves exchanges of both charged and neutral pions.

However, it is formed that it is not possible to explain all features of the nucleon-nucleon force in terms of exchange of pions only. The reason is that the pion is only one of many mesons. It leads to the long range part of the nucleon-nucleon force. For ranges shorter than 2 m, other meson exchanges should be included. There are several scalar, pseudoscalar and vector mesons that occur with a mass less than 1 GeV. For instance, the η mesons, the ρ mesons and the ω meson, when exchanged, yield a potential of range

$$\lambda_\eta = \frac{\hbar}{m_\eta\, c} = 0.360 \text{ fm} \qquad (m_\eta c^2 = 549 \text{ MeV})$$

$$\lambda_\rho = \frac{\hbar}{m_\eta\, c} = 0.258 \text{ fm} \qquad (m_\rho c^2 = 763 \text{ MeV})$$

$$\lambda_\omega = \frac{\hbar}{m_\omega\, c} = 0.252 \text{ fm} \qquad (m_\omega c^2 = 783 \text{ MeV})$$

Further, it is known that the exchange of ω meson gives rise to a repulsive core. If the exchange of all known mesons with their corresponding coupling constants and the exchange of two pions are included, the experimentally determined nucleon-nucleon potential can be explained satisfactorily up to 2 or 3 GeV. Thus the fundamental idea of Yukawa that the nucleon-nucleon potential arises from exchange of hadronic quanta is confined.

6.5 charge independent and charge symmetry of nuclear forces

Since neutron as well as proton must be bound. This means that the nuclear interaction between as well as protons. The protons or one proton and one neutron are basically same. For e.g from the analysis of p-p and n-p scattering. Scientists have concluded that the nuclear part is essentially the same in both cases. Also the facts that

a) Light nuclei are composed of equal no of protons and neutrons.

b) Binding energy/nucleon is approximately constant

c) Mass difference of mirror nuclei can be accounted for the difference in coulomb energy atoms, indicate that the nuclear interaction is charge independent. Because of this property, P and n are considered in so far.

6.6 Normalization of deuteron wave function:

The constants A and B appearing in the potential function of deuteron are calculated and used to obtain the r m s electromagnetic radius of the deuteron.

Applying the normalization condition to the deuteron wave function we have

$$A^2\int_c^{c+b} sin^2 k(r-c)dr + B^2\int_{c+b}^{\infty} e^{-2\alpha\hbar}\, dr = 1$$

Integrating and rearranging we get

$$A^2 = \frac{2k}{1+\alpha b}$$

And $B^2 = \frac{2\alpha\,(sin^2 kb)e^{2\alpha\,(c+b)}}{(1+\alpha b)}$

Let $\prec r^2 d \succ$ be the average of the square of the proton to centre of mass distance.

This is half the separation between the proton and the neutron. Then

$$\prec r^2 d \succ = \frac{A^2}{4}\int_c^{c+b} r^2 sin^2 k(r-c)dr + \frac{B^2}{4}\int_{c+b}^{\infty} e^{-2\alpha r}\, dr$$

Solving we get

$\prec r^2 d \succ$=2.22F which is close to the experimental value of 2.1 F.

6.7 LET US SUM UP:

- The nuclear force. is very strong at distances of the order of nuclear size there must be compensate the coulomb repulsion between two protons.
- The nucleon-nucleon potential should be a function of the three dynamical variables, the relative position coordinate r and the spins σ_1 and σ_2 of the two nucleons and iso spin.
- The neutron as well as proton must be bound.

6.8 Review questions:

1. How is deuteron wave function normalized?

2. What are tensor forces?

3. Discuss the theory on nucleon-nucleon potential.

4. Write the theory about the charge independence and charge symmetry of nuclear forces.

5. Write an essay about the nuclear forces.

6.9 Further reading:

1. Nuclear and particle physics –An introduction- Brian R Martin.

2. Nuclear Physics- V.Devananthan, Narosa Publishers, New Delhi.

UNIT VII PARTIAL WAVE ANALYSIS

Structure:

7.1 Introduction:

Partial wave analysis, in the context of quantum mechanics, refers to a technique for resolving scattering problems by decaying each wave into its constituent angular momentum components and solving using boundary conditions. Yukawa explained that the meson which is nothing but combination of photons and gravitons. It has finite mass and also slightly higher than the electron mass.

7.2 Objectives: Method of partial wave analysis and phase shifts are discussed- Effective range theory is deliberated- n-p scattering at low energies is discussed- Yukawa's meson theory of nuclear forces is discussed.

7.3 Method of partial wave analysis:

When a beam of particles strike a target, each nucleus in the target acts as an interaction centre. Part of the incident beam is absorbed while part of it is scattered which is shown in Fig.7.1. The cross section associated with the process of scattering is known as scattering cross section. The cross section associated with the process of reaction is known as reaction cross section. Hence the total cross section is $\sigma_{tot}=\sigma_{sc}+\sigma_{a}$. The cross section can be obtained using partial wave analysis.

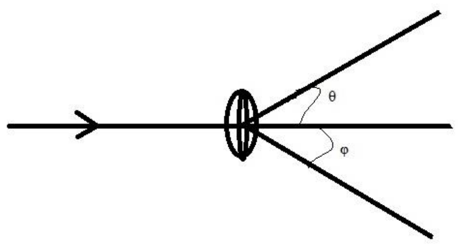


Fig 7.1

For simplicity, let us consider the problem in the centre of mass co-ordinate system. Let a beam of particle be incident along the z-direction. The wave function for the incident particle is

$\Psi_{inc}=e^{ikz}=e^{ikr\,cos\theta}$

Where the wave number k is related to the reduced mass 'm' and energy as

$K^2=\frac{2mE}{\hbar^2}$

The incident plane wave can be expanded as

$\Psi_{inc}=e^{ikr\,cos\theta}=\sum_{l=o}^{\infty}B_2(\mathrm{r})\,y_{r\,l}\mathrm{O}^{(\theta)}$ ---------------------------(1)

The coefficient $B_l(r)$ is expressed in spherical Bessel function $j_e(kr)$ as

$$B_l(r)=i^l\sqrt{4\pi(2l+1)}\,j_l(kr)$$

Equ (1) becomes

$\Psi_{inc}=\sum_{l=0}^{\infty}i^l\sqrt{4\pi(2l+1)}\,j_l(kr)\,y_l\,O^{\theta}$

Considering the first order spherical Bessel function we have $j_l(kr)=\frac{\sin kr}{kr}$

$\Psi_{inc}=\sum_{l=0}^{\infty}i^l\sqrt{4\pi(2l+1)}\,\frac{\sin kr}{kr}y_l\,O^{\theta}$

since $\sin kr=(\frac{e^{ikr}-e^{-ikr}}{2i})$, the incident wave function can be given as

$\Psi_{inc}=\sum_{l=0}^{\infty}i^{l+1}\sqrt{4\pi(2l+1)}\,(\frac{e^{ikr}-e^{-ikr}}{2kr})\,y_l\,O^{\theta}$

When a particle passes through a nuclear potential, it suffers a phase change of $l\pi/2$ where 'l' is the orbital angular momentum quantum number. Hence

$\Psi_{inc}=\sum_{l=0}^{\infty}i^{l+1}\sqrt{4\pi(2l+1)}\,(\frac{e^{i(kr-\frac{l\pi}{2})}-e^{-i(kr-\frac{l\pi}{2})}}{kr})\,y_l\,O^{\theta}$ --------(2)

The first term of equ.(2) represents the incoming wave at any arbitrary point where as the second term represents the outgoing part of the wave.

For l=0 neutrons, equ (2) becomes

$\psi_{inc}=\sum\frac{\sqrt{4\pi}}{kr}[\,e^{-i(kr-\frac{l\pi}{2})}-e^{i(kr-\frac{l\pi}{2})}]\,)\,y_l\,O^{\theta}$

7.4 Determination of the Phase shift:

The potential experienced by incident neutron can be considered as a square well potential as shown below.

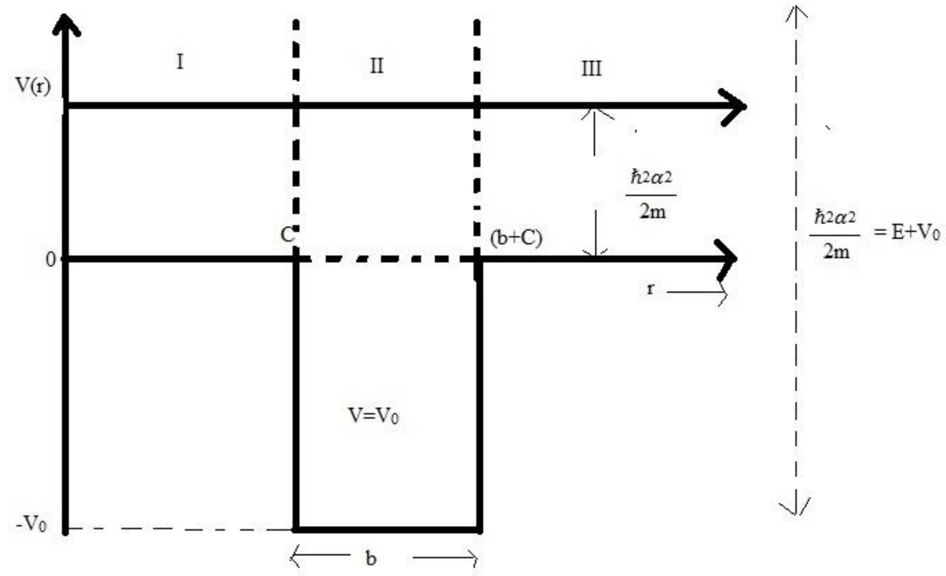


Fig.7.2

The potential is described as

V= 0 for r < c (regionI)

V= -V_0 for c < r < b+c (region II)

V= 0 for r > (b+c) (region III)

The S.E for the region II is

$$\frac{d^2u2}{dr^2}+\frac{2m}{\hbar^2}(E+V_0)\,u_2=0$$

The solution is u_2= A sink(r-c)

Where A is the normalization constant and $k^2=\frac{2m}{\hbar^2}(E+V_0)$.

For the region III, the S.E is

$$\frac{d^2u_3}{dr^2}+\frac{2m}{\hbar^2}\,u_3=0$$

The solution is u_3= B sin(αr+δo)

Where $\alpha^2=\frac{2mE}{\hbar^2}$, 'B' is the normalization constant and δo is the phase difference.

Applying the boundary conditions namely at r=b+c

(i) $u_2 = u_3$ and

(ii) $\frac{du_2}{dr}=\frac{du_3}{dr}$

Applying the first condition

A sin k (b+c-c)= B sin[α(b+c)+ δo]

A sin kb= B sin [α (b+c) +δo] ------------------------(2)

Applying the second condition

Ak cos kb= Bα cos[α(b+c) +δo] ------------------------------(3)

Equ (3)/ Equ (2) gives

k cot kb= cot (α (b+c) +δo)

With the values of 'b' V, E and m, values of k and α can be obtained and used to determine the phase shift δo, using δo, the scattering cross section can be obtained.

Scattering length: The concept of scattering length was introduced by Fermi and Marshall in order to explain the neutron-proton interaction at low energies. The total wave function in a n-p scattering is

$\psi= e^{i\sqrt{0}} \frac{\sin(kr+\delta o)}{kr}$

When E→0, k→ 0. In that case δo becomes either zero or π

Lt k→0 $e^{i\delta o} \frac{\sin(kr+\delta o)}{kr} = \frac{kr+\delta o}{kr}$

7.5 Effective range theory:

For the low energy n-p scattering, the scattering cross section depends on the phase shift, which depends on the shape of the potential. For potentials of different shapes, the phase shifts calculation becomes much difficult. But it is possible to expand δo in a power series of k [$k^2= \frac{2mE}{\hbar^2}$]. In which δo does not depend on the shape of the potential. The expansion contains two terms, namey scattering length (a) and effective range (r_0)

Let u be the solution of the radial wave equation for l=0 states inside and outside the potential well, with an energy E.

Let us define a new wave function v, which is identical to 'u' outside the potential well. But inside the well it is extrapolated such that v=1 at r=0

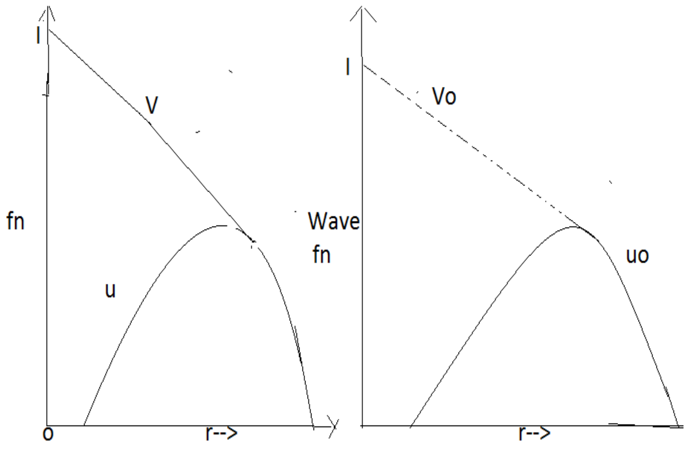


Fig. 7.3

Both'u' and'v' are the normalized wave function. Hence the form of 'v' can be

$$v=\frac{\sin(kr+\delta o)}{sin\delta o}$$

Let 'u_o' be the solution of a wave function with energy E_0 where $E_0\rightarrow 0$
Hence u and u_0 be the solution of the S.E

$\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2}$+Vu=Eu --------------------------------------(1)

and $\frac{-\hbar^2}{2m}\frac{d^2uo}{dr^2}+Vu_0=E_0u_0=0$ --------------------------(2)

Where V is the constant potential.

Equ (1) $\times u_0$ gives

$$\frac{-\hbar^2}{2m}uo\frac{d^2u}{dr^2}+Vuu_0=Euu_0 \quad \text{------------------------------(3)}$$

Equ (2) $\times$u gives

$$\frac{-\hbar^2}{2m}u\frac{d^2uo}{dr^2}+Vuu_0=E_0u_0u \quad \text{----------------------------------(4)}$$

Equ(3)- Equ(4) gives

$$\frac{-\hbar^2}{2m}(u\frac{d^2uo}{dr^2}-uo\frac{d^2u}{dr^2})=(E-E_0)\,uu_0$$

$$u\frac{d^2uo}{dr^2}-u_0\frac{d^2u}{dr^2}=\frac{2m(E-Eo)uuo}{\hbar^2}$$

$$\frac{d}{dr}(uu_0'-u_0u')=(k^2-k_0^2)\,uu_0 \quad \text{-----------------(5)}$$

And $k^2=\frac{2mE}{\hbar^2}$ $\quad k_0^2=\frac{2mE_o}{\hbar^2}$

If ‘v_0’ is the solution of the radial wave equation for the region outside a constant potential for l=0 state corresponding to the energy E_0 then, we can write

$\frac{d}{dr}(vv_0'-v_0v') = (k^2-k_0^2)\ vv_0$ --------------------------(6)

Where outside the potential u=v and $u_0=v_0$

But inside the potential $u=u_0=0$ at r=0 and

$v=v_0=1$ at r=0

Subtracting equ (5) from equ (6) and then integrating

$\int_0^\infty \frac{d}{dr}(vv_0'-v_0v')-\frac{d}{dr}(uu_0'-u_0u')\,dr= \int_0^\infty(k^2-k_0^2)\,(vv_0-uu_0)dr$

$(vv_0'-v_0v'-uu_0'+u_0u')_0^\infty= \int_0^\infty(k^2-k_0^2)(vv_0-uu_0)dr$

$[\,uu_0'-u_0u'-uu_0'+u_0u']-[1v_0'-v'-0u_0'+0u']_{r=0}$

With u=v

$= \int_0^\infty k^2-k_0^2)\,(vv_0-uu_0)dr$

$-(v_0'-v')_{r=0} = \int_0^\infty k^2-k_0^2)\,(vv_0-uu_0)dr$

we know that $v = \frac{\sin(kr+\delta o)}{sin\delta o}$

$v'=k\frac{\cos(kr+\delta o)}{sin\delta o}$

at r=0

$v'= k\cot\delta o$

$v_0' = \frac{\sin(kor+\delta o)}{sin\delta o}$

$v_0'= \frac{k_o \sin kor+\delta o)}{sin\delta o}$

At r=0, equ (7) becomes

$-(ko\cot\delta o-k\cot\delta o) = (k^2-k_o^2)\int_0^\infty(vv_0-uu_0)\,dr$ ------------------(8)

At moderate energies $u=u_0$

$v=v_0$

At $t_0\to0$ then $k_0\to0$

$K_0\cot\delta o= -\frac{1}{a}$ where a → scattering length

Hence equ (8) be

$\frac{1}{a}$ +k cotδo= k^2 $\int_0^\infty v_0^2 - u_0^2)$ dr ---------------------(9)

Let $\int_0^\infty (v_0^2 - u_0^2)$ dr = $\frac{r_0}{2}$ where r_0 →effective range

Equ (9) become

$$\text{kcot } \delta_0 = \frac{1}{\alpha} + \frac{1}{2} r_0 k^2$$

$$\cot \delta_0 = -\frac{1}{kc} + \frac{1}{2} r_0 \text{ k} \qquad \text{---------------------------------(10)}$$

Equation (V) gives the value of δo, which is independent of the shape of the potential.

The scattering cross section for l=0 states

$$\sigma_{sc,0} = \frac{4\pi sin^2 \delta o}{k^2}$$

$$= \frac{4\pi}{k^2} \frac{1}{cosec^2 \delta o} = \frac{4\pi}{k^2(1+cot^2 \delta o)}$$

$$= \frac{4\pi}{k^2 + k^2 cot^2 \delta o}$$

$$= \frac{4\pi}{k^2 + (-\frac{1}{a} + \frac{1}{2} r_0 k^2)^2}$$

$$= \frac{4\pi r^2}{\alpha^2 k^2 + (1 - \frac{1}{2} a r_0 k^2)^2}$$

$$\sigma_0 = \frac{4\pi\alpha^2}{\alpha^2 k^2 + (1 - \frac{1}{2} a r_0 k^2)^2}$$

But the total cross section is the weight average of singlet and triplet states

Hence

$$\sigma_{\text{tot},0} = \frac{3}{4}\, \sigma_{t,0} + \frac{1}{4}\, \sigma_{r,0}$$

$$\sigma_{\text{tot},0} = \frac{3\pi a_1^2}{a_t k^2 + [1 - \frac{1}{2} a_t r_{0,t} k^2]^2} + \frac{\pi a_0^2}{a_s k^2 + [1 - \frac{1}{2} a_s r_{0,s} k^2]^2}$$

where a_t,a_0 can be determined through low average scatter experiments.

7.6 Low energy n-p scattering:

Consider a neutron moving along the z-direction with a constant velocity ‘v’ and incident on a proton. The incident particle can be described by a plane wave function.

$\psi_{\text{inc}} = e^{ikz} = e^{ikr\, cos\theta}$

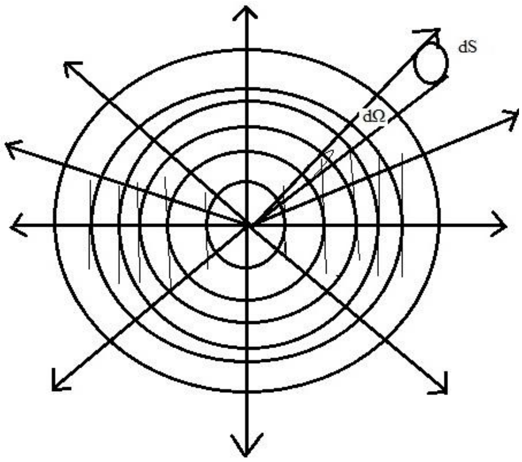


Fig.7.5

Where k is the wave vector related to the total energy 'E' of the system

$k^2 = \frac{2mE}{\hbar^2}$, where 'm' is the reduced mass of the system.

Since $|\psi_{inc}|^2 = 1$, the incident beam contains only one particle per unit volume

The incident flux=$|\psi_{inc}|^2$ v=v

Let us consider l=0 neutrons. The neutrons wave function becomes

$\psi_0 = \sqrt{4\pi}\,\frac{sinkr}{kr}\,\frac{1}{2\sqrt{\pi}}$ where $y_{\theta,}0^{(\theta)} = \frac{1}{2\sqrt{\pi}}$

$\psi_0 = \frac{sinkr}{kr} = \frac{(e^{ikr} - e^{-ikr})}{2ikr}$ --------------------------------(3)

When the incident particle enters into the scattering potential of the nucleus, it undergoes a change in its phase occurs. Let $\delta 0$ is the phase change produced which is shown in Fig 7.5

Now equ.(3) becomes

$\psi_0 = \frac{[e^{i(kr+2\delta o]} - e^{-ikr}]}{2ikr}$

$\psi_0 = \frac{[e^{i(kr+2\delta o]} - e^{-ikr}]}{2ikr}$

$\psi_0 = e^{i\delta o}\ \frac{\sin(kr+\delta o)}{kr}$

The total wave function in the presence of scattering potential is obtained by adding the difference of equ (3) and equ (4) with ψ_{inc}

$\psi = \psi_{inc} + \frac{[e^{i(kr+2\delta o]} - e^{-ikr}]}{2ikr}$

$= e^{ikz} + \frac{[e^{i(kr+2\delta o]} e^{i\delta o} - e^{+ikr} e^{-i\delta o}]}{2ikr}$

$\psi = e^{ikz} + e^{i(kr+\delta o)} \frac{\sin\delta o}{kr}$

This is the wave function for the scattered particle. The number of particles scattered per second is obtained by integrating the flux over a sphere of radius r.

$$Nsc = \int |\psi_{sc}|^2\, 4\pi v r^2 dr$$

$$= \frac{\sin^2\delta o \times 4\pi v r^2}{k^2 r^2}$$

$$= \frac{\sin^2\delta o \times 4\pi v}{k^2}$$

The scattering cross section is defined as the ratio of scattered flux to incident flux.

$$\sigma_{sc,0} = \frac{\sin^2\delta o \times 4\pi v}{k^2 v}$$

$$\sigma_{sc,0} = \frac{\sin^2\delta o \times 4\pi}{k^2}$$

For other values of l, we have

$$\sigma_{sc,0} = \frac{\sin^2\delta o \times 4\pi}{k^2}.\ (2l+1)$$

7.7 Yukawa's meson theory of nuclear forces:

Yukawa prosed that nuclear forces are due to the exchange of particles between the nucleons. In electromagnetic field, exchange of photons takes place. In gravitational field, exchange of gravitations takes place. In the same way in the meson field by which nuclear force is taking place, exchange of mesons takes place. Meson has a finite mass slightly higher than the mass of an electron, whereas photons and gravitons are massless particles. When a nucleus exerts a force on another, meson is created and a loss of kinetic energy is associated with it.

If ϕ is a scalar potential, then Maxwell's electromagnetic equation becomes

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})\phi = 0 \quad \text{-------------------(1)}$$

The equation (1) is valid for free space, for a static field, in the presence of charges equation (1)

Becomes

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})\phi = -\frac{l}{\varepsilon_o} \quad \text{----------------(2)}$$

Where l is the charge density and εo is the permittivity of the free space. Equation can be written as

$\nabla^2\varphi = -\frac{l}{\varepsilon_0}$ --------------------------(3)

Integrating equation (3), the solution becomes

$\varphi=\frac{q}{4\pi\varepsilon_0 r}$

Where ‘r’ denotes the distance of the charge from the point at which the scalar potential is considered.

The momentum operator and energy operator are given as

$P = -i\hbar\frac{\partial}{\partial x}$

$E = i\hbar\frac{\partial}{\partial t}$

Momentum and energy is related as

$E^2= p^2c^2+m^2c^4$

For a particle of zero rest mass equ. (4) becomes

$-p^2+\frac{E^2}{c^2}=0$

For the non-zero rest mass equ (4) becomes

$-p^2+\frac{E^2}{c^2} - m^2c^2=0$

Using momentum operator (P) and energy operator (E) equation (5) becomes

$(\nabla^2-\frac{m^2c^2}{\hbar^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\Phi= 0$

Where φ is the scalar meson field.

Splitting equ (6) into time dependent and time independent parts and equating the time independent part equation to zero we get

$(\nabla^2- \frac{m^2c^2}{\hbar^2})\ \varphi=0$

Let $\frac{mc}{\hbar}=\mu$

$(\nabla^2-\mu^2)\varphi=0$

The solution of the equation (7) is

$\varphi=g\frac{e^{-\mu r}}{r}$

Where ‘g’ is a constant which plays the role of the charge and depends on the source at the origin.

The meson potential becomes

V(r) =gφ= g$^2 \frac{e^{-\mu r}}{r}$, the plot of V(r) with 'r' is shown below

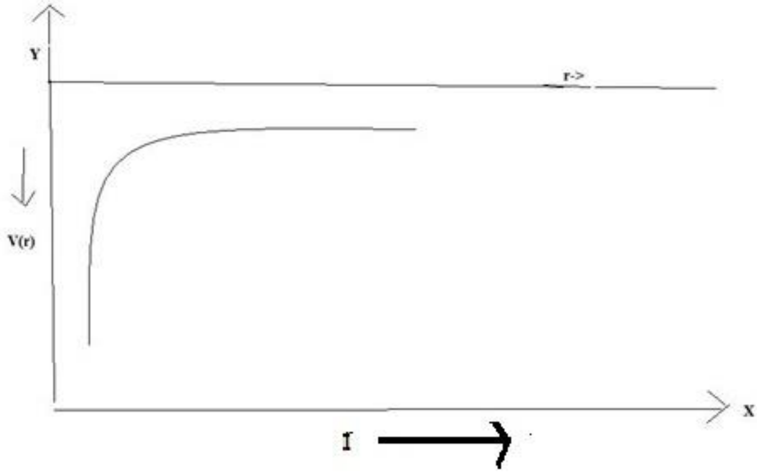


Fig.7.6

7.7.1 Determination of mass of meson:

For the creation of a meson in the real state, sufficient energy must be supplied, which comes from the kinetic energy of the colliding nucleons. If m_π is the mass of a meson, a meson is released from a nucleon when an energy of m_0c^2 is supplied to it.

i.e., $\nabla.E=m_\pi c^2$

If ∇t is the duration for which the meson exists then by Heisenberg uncertainty principle

$\nabla.E\ \nabla t=\frac{h}{2\pi}$

$\nabla t= \frac{h}{2\pi \nabla E}$

If r_o is the distance traveled with in this time then

$r_o= c\nabla t= \frac{ch}{2\pi \nabla E} =\frac{ch}{2\pi m_\pi c^2}= \frac{h}{2\pi m_\pi c}$

$= \frac{\hbar}{m_\pi c}$ where $\hbar = \frac{h}{2\pi}$

If r_0 is takes as the range of the nuclear force which is equal to 1.4 F, then $m_\pi=\frac{\hbar}{r_0 c}$= 270 times the mass of an electron.

Theses mesons are known as π-mesons or pions. Later on three kinds of pions such as π+,π^0 and π^- were detected. π^+, π^- are the charged pions whereas π^0 is the neutral pion. The force between two protons or two neutrons is carried by neutral pions whereas force between a proton and a neutron produced due to the exchange of charged pions.

Yukawa's meson theory helps in predicting the

(i) the existence of pions

(ii) mass of the pions

(iii) spin and parity of the pions.

7.8 LET US SUM UP:

- ❖ For the low energy n-p scattering, the scattering cross section depends on the phase shift, which depends on the shape of the potential
- ❖ Yukawa prosed that nuclear forces are due to the exchange of particles between the nucleons.
- ❖ Meson has a finite mass slightly higher than the mass of an electron, whereas photons and gravitons are massless particles

7.9 Review questions:

1. On the basis of partial wave analysis obtain an expression for the scattering cross section for the low energy n-p scattering.
2. Discuss the effective range theory in detail.
3. What is partial wave analysis? Explain
4. Bring out the salient features of Yukawa's theory of mesons.
5. How is the mass of π-meson determined from Meson's theory of nuclear forces?

7.10 Further reading:

1. Physics of the Nucleus- Gupta and Roy Arunabha Publisher, Kolkata
2. Modern Atomic and Nuclear physics- A.B.Gupta.

BLOCK III REACTION CROSS SECTIONS AND NUCLEAR REACTORS

UNIT VIII REACTION CROSS SECTION

Structure:

8.1 Introduction:

The nuclear cross section of a nucleus is used to characterize the possibility that a nuclear reaction will occur. The concept of a nuclear cross section can be enumerated physically in terms of "characteristic area" where a larger area means a larger possibility of interaction. The standard unit for measuring a nuclear cross section (denoted as σ) is the barn, which is equal to 10^{-28} m² or 10^{-24} cm². Cross sections can be measured for all possible interaction routes together, in which case they are called total cross sections, or for specific processes, distinguishing elastic scattering and inelastic scattering.

8.2 Objectives: Nuclear cross sections are discussed- Compound nucleus formation and breakup are deliberated- Resonance scattering cross section is discussed.

8.3 Nuclear cross section:

When a beam of collimated mono-energetic protons or neutrons strikes a target containing atoms or nuclei, interaction takes place. As a result of the interaction the following nuclear processes may take place.

1. The incident particle may be simply deviated from its path. This is known as scattering. There are two types of scattering namely elastic scattering in which, no loss of energy occurs between the incident and outgoing particles. In the process of inelastic scattering, there will be a change in the energy of the outgoing particle.

2. The incident particle may be completely absorbed by target nuclei without emission of any particle. This refers to as radioactive capture.
3. The incident particle and outgoing particle may differ from each other. This process is known as nuclear reaction.

The cross section of a nuclear reaction is defined as the effective area offered by the nucleus for the incoming particle for a particular type of process of take place. In other words cross section (σ) can be defined as the ratio of the number of given type of events per unit time per nucleus to the number of projectile particles per unit area per time. The cross section associated with the process of scattering is known as scattering cross section (σ_{sc}). The cross section associated with the process of reaction, it is known as reaction cross section (σ_r) or absorption cross section (σ_a). If there is absorption as well as scattering, the total cross section is the sum.

$$\sigma_{tot} = \sigma_{sc} + \sigma_a$$

where σ_{sc} and σ_a denote the scattering cross section and absorption(reaction cross section. This cross section has the dimension of area. It is expressed in barn. (1 barn=10^{-28} m^2)

8.4 Expression for nuclear cross sections:

Let a beam of neutron be incident on a target nucleus considered at any arbitrary point. The incident beam can be considered as a plane wave function (ψ_{inc}). For larger values of r, ψ_{inc} becomes,

$$\psi_{inc} = e^{ik.z} = e^{ikr.cos\theta}$$

$$= \frac{\sqrt{\pi}}{kr} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^{(l+1)} [e^{-ikr} - e^{ikr}]\, Y_{l,0}(\theta)$$

Where $K^2 = \frac{2mE}{\hbar^2}$, where E is the energy of the incident particle.

Because of the presence of nuclear potential the wave function for the incident neutron is modified as,

$$\psi_{inc} = \frac{\sqrt{\pi}}{kr} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^{(l+1)} [e^{-i(kr-\frac{l\pi}{2})} - e^{i(kr-\frac{l\pi}{2})}]\, Y_{l,0}(\theta) \quad \text{----(1)}$$

Where 'l' represents the angular momentum state of the neutron. The first term of equ.(1) represents the incoming part of the incident wave while the second of equ.(1) describes the outgoing part of the wave. Since $|e^{ik.z}|$=1, there is only one particle per unit volume in the |incident beam. If v is the velocity of the incident neutron then, the incoming flux= $|\psi_{in}|^2$ v=v --------(2)

Let the neutron be scattered by the target nucleus. Let the amplitude of the scattered wave be changed by a factor η_i, without any change in its phase, where $|\eta_i| \leq 1$.

The total wave function becomes,

$$\psi_{tot} = \frac{\sqrt{\pi}}{kr}\sum_{l=0}^{\infty}\sqrt{2l+1}\, i^{(l+1)}\, [e^{-i(kr-\frac{l\pi}{2})} - \eta_i e^{i(kr-\frac{l\pi}{2})}]\, Y_{l,0}(\theta) \quad \text{-----------(3)}$$

The wave function of the scattered particle becomes,

$$\psi_{sc} = \psi_{tot} - \psi_{inc}$$

$$= \frac{\sqrt{\pi}}{kr}\sum_{l=0}^{\infty}\sqrt{2l+1}\, i^{(l+1)}\, [e^{-i(kr-\frac{l\pi}{2})} - (1-\eta_i]\, Y_{l,0}(\theta) \quad \text{----------(4)}$$

The scattered flux = $|\psi_{sc}|^2.v$

The total number of particles scattered per second over a shell of radius r can be obtained by integrating the flux over the angles.

$$N_{sc,J} = \frac{\pi}{k^2r^2}(2l+1)\,|1-\eta_l|^2.v \int |Y_{l,0}(\theta)|^2 .r^2 sin^2\theta\, d\tau$$

$$= \frac{\pi}{k^2r^2}(2l+1)\,|1-\eta_l|^2.v.r^2$$

$$= \frac{\pi}{k^2}(2l+1)\,|1-\eta_l|^2.v$$

Outgoing flux= $\frac{\pi}{k^2}(2l+1)\,|1-\eta_l|^2.v$

The partial scattering cross section $\sigma_{sc,J}$ is defined as the ratio of the outgoing flux to the incident flux.

$$\sigma_{sc,J} = \frac{\pi}{k^2}(2l+1)\,|1-\eta_l|^2 \quad \text{-----------------(5)}$$

Based on similar lines, the partial reaction cross section associated with the interaction becomes,

$$\sigma_{r,J} = \frac{\pi}{k^2}(2l+1)\,(1-|\eta_l|)^2 \quad \text{-----------(6)}$$

Since there is absorption, $|\eta_l| < 1$.

The partial reaction (absorption) cross section becomes,

$$\sigma r = \sum_{l=0}^{\infty}\sigma_{r,J}$$

Equation (5) and (6) are valid only at larger distances, and is invalid at shorter distances.

Let us consider l=0 neutrons. Then,

$$\psi_{tot} = \frac{\sqrt{\pi}}{kr}\, I\, [e^{-ikr} - \eta_0 e^{ikr}]\, Y_0,0^{(\theta)}$$

$= \frac{u_0}{r} Y_{0},0^{(\theta)}$

Where $u_0 = i.\frac{\sqrt{\pi}}{k} [e^{-ikr} - \eta_0 e^{ikr}]$ is the radial wave function. This relation is valid at both larger and shorter distances.

Let us define f_0 as logarithmic derivative as,

$$f_0 = \left(\frac{r}{u_0} . \frac{du_0}{dr}\right)_{r=R} \quad \text{---------------------------------(7)}$$

where 'R' is the radius of the nucleus, and f_0 is the logarithmic derivative.

We know that,

$$u_0 = \frac{i\sqrt{\pi}}{k} (e^{-ikr} - \eta_0 e^{ikr}) \quad \text{--------------------------------(8)}$$

Differentiating Equ (8), substituting in Eq.(7) and rearranging we get,

$$\eta_0 = \left(\frac{f_0 + ikR}{f_0 - ikR}\right) e^{-2ikR} \quad \text{------------------------(9)}$$

Using equ (9), $\sigma_{sc,0}$ and $\sigma_{r,0}$ can be obtained with the help of equ (5) and (6).

8.5 Theory of compound nucleus:

When an incident particle approaches a target nucleus, it gets absorbed by the target nucleus. This forms a compound nucleus in the excited state. This is known as formation process. The compound nucleus which is in the excited state decays with the emission of a particle. This is known as decay process. The mode of disintegration of the compound nucleus depends only on the specific way in which it has been formed. Since the decay process is independent of the formation process, the reaction cross section is given as,

$$\sigma(p,q) = \sigma_c (E_c,p)\, G_c (E_c,q) \quad \text{-----------------(1)}$$

where p and q are the angular momentum states of the incident and emitted particles respectively. E_c is the excitation energy of the compound nucleus, σ_c and G_c are the probability for (cross sections) the formation process and decay process respectively. In order to calculate the cross section associated with the formation process, the following assumptions are made.

1. The nucleus has a well-defined surface with a radius R.
2. Inside the nucleus, the potential is negative, ie., $v = -v_0$

Since the energy of the incident particle is shared by the entire nucleus in the target, the probability of a particle re-emitted without loss of energy becomes negligible. If the outgoing particle has full energy, then the process is known as elastic scattering and if the particle does not at all enter into the nuclear potential, it is known as potential scattering.

For l=0 neutrons, the Schrodinger wave function for a particle inside the nucleus is,

$\frac{d^2u_0}{dr^2}+\frac{2m}{\hbar^2}(E+V_0)u_0=0$

$\frac{d^2u_0}{dr^2}+\alpha^2u_0=0$ -------------------(2)

Where α is the wave vector inside the nucleus, given as

$\alpha^2+\frac{2m}{\hbar^2}(E+V_0)$.

Assuming that the compound elastic scattering is negligible, within the surface of the nucleus, the incoming wave is represented by the solution of equ.(2) as,

$u_0 = e^{-i\alpha R}$

Let us define logarithmic derivative f_0 as,

$f_0 = \frac{r}{u_0}\left(\frac{du_0}{dr}\right)_{r=R}$

$= \frac{R}{e^{-i\alpha R}}(-i\alpha)\, e^{-i\alpha R}$

$f_0 = -i\alpha R$

But $\eta_0 = \left(\frac{f_0+ikR}{f_0-ikR}\right) e^{-2ikR}$

Substituting for f_0, we get

$\eta_0 = \left(\frac{-i\alpha R+ikR}{-i\alpha R-ikR}\right) e^{-2ikR}$

Then $|\eta_0| = \frac{\alpha-k}{\alpha+k}$ --------------------------(3)

The reaction cross section for l=0 neutron is defined as,

$\sigma_{r,0} = \frac{\pi}{k^2}(1-|\eta_0|^2)$ -------------------------(4)

Substituting for $|\eta_0|$ in equ.4 and rearranging,

$\sigma_{r,0} = \frac{4\pi\alpha}{k(\alpha+k)^2}$ --------------------------------(5)

If the energy of the incident particle approaches zero, $\hbar^2k^2$ approaches zero and hence $\sigma_{r,0} = \infty$.

Hence this theory is invalid at very small energies. If the incident beam consists of neutrons of different angular moment states then the total cross section becomes,

σ_c (E_c,p) = $\sum_{l=0}^{\infty} \sigma_{n,l}$ -----------------------(6)

The above represents the cross section associated with the formation process.

The nucleus can decay in many numbers of final channels. Let us assume that nucleus be breaking up through a channel q with an energy E_q and a decay constant λq.

By Heisenberg's uncertainty principle

$\Delta E.\Delta\varepsilon = \hbar$

$\Delta E\tau = \hbar$

$=\Gamma(\frac{1}{\lambda})=\hbar$

$\Gamma=\lambda\hbar$ where $\Gamma=\Delta E$ and $\tau=\Delta\varepsilon$.

Hence $\Gamma_q = \hbar\lambda e = \frac{\hbar}{\tau_q}$ = Level width. Considering all the possible channels the total level width is $\Gamma = \sum_q \Gamma_q$.

The probability of decay is G_c (E_c,q) = $\frac{\Gamma_q}{\Gamma}$.

According to reciprocity theorem

$\frac{k_p^2\sigma_c(p)}{\Gamma_p} = \frac{k_q^2\sigma_c(q)}{\Gamma_q} =$ ….. U(E_c) ---------------(7)

Where U(E_e) is the function of the excitation energy of the nucleus and this does not depend on the channels. Then,

$$G_e(E_e,q) = \frac{\Gamma_q}{\sum_r \Gamma_\gamma}$$

$$= \frac{\lambda_q}{\sum_\gamma \lambda_\gamma}$$

$$G_e(E_e,q) = \frac{k_e^2\sigma_c(q)}{\sum_r k_\gamma^2\sigma_c(\gamma)}$$

Hence σ_c(p, q) can be obtained.

8.6 Resonance scattering: Breit-wigner one level formula

The interaction of neutrons with nuclei depends on the energy of the incident particle. If the energy of the incident neutron is below the lower excitation energy of the compound nucleus (E_c), the elastic scattering takes place. If the energy of the neutron is raised, the cross section shows narrow peaks at definite excitation energies. These peaks are known as resonance scattering. They occur for slow neutrons in nuclei of middle and high mass numbers (A). For protons and alpha particles it occurs for A$\leq$ 30. For simplicity let us consider l=0 neutrons in a square well potential. The wave function of the particle varies with distance r.

The logarithmic derivative f_0 is given as,

$$f_0 = \left(\frac{r}{u_0}\ \frac{du_0}{dr}\right)_{r=R}$$

But u_0 is the solution of the Schrodinger wave equation given as,

$$u_0 = \frac{\sqrt{\pi}}{kr}\left(e^{-ikr} - \eta_0 e^{ikr}\right) Y_{l,0}^{(\theta)} \qquad \text{where } k^2 = \frac{2mE}{\hbar^2}$$

As the energy of the incident neutron tends to zero, u0 and hence f0 tends to zero, f_0 decreases from positive value to negative value through zero. This behavior of f0 can be mathematically represented as,

$$f_0 = -\alpha R \cot\alpha R\ \alpha R$$

where α is the wave vector inside the potential well such that $\alpha^2 = \frac{2m}{\hbar^2}$ $(E+V_0)$.

Let f_0 goes to zero at an energy Es such that, f_0 = -a (E-Es) where 'a' is a constant. If we assume f_0 as a real quantity, then $|\eta_0|=1$. Hence $\sigma_{r,0}$ becomes zero as,

$$\sigma_{r,0} = \frac{\pi}{k^2}\left[1-|\eta_0|^2\right]$$

Hence an imaginary term is added such that f_0= -a (E-Es)-i_b where b is a constant.

Let η_0 be the factor by which the amplitude of the scattered wave is scaled down. η_0 is related to f_0 as,

$$\eta_0 = \frac{f_0+ikR}{f_0-ikR}\,.e^{-2ikR}$$

Then, $|1-\eta_0| = \left[\,1-\frac{f_0+ikR}{f_0-ikR}\,.e^{-2ikR}\right]$

$$= \left|\frac{1}{e^{2ikR}}\right|\left|e^{2ikR}-\frac{f_0+ikR}{f_0-ikR}\right|$$

$$= \left|e^{2ikR} - \frac{f_0-ikR+2ikR}{f_0-ikR}\right|$$

$$= \left|e^{2ikR} - 1 - \frac{-2ikR}{f_0-ikR}\right|$$

$$= |A_{pot} + A_{res}|$$

Where $A_{pot} = e^{2ikR}$-1

$= e^{ikR}|e^{ikR} - e^{-ikR}|$

$A_{pot} = 2ie^{ikR}$ sinkR ----------------------------(1)

$A_{res} = -\frac{2ikR}{f_0 - ikR}$ ----------------------------(2)

If $A_{pot} <<< A_{res}$, then

$|1-\eta_0| = |\frac{-2ikR}{f_0 - ikR}|$

$= \frac{-2ikR}{-\alpha(E-E_0) - ib - ikR}$

$= \frac{-2ikR}{(E-E_0) + i(\frac{b}{a} + \frac{kR}{a}}$

But, $\sigma_{sc,0} = \frac{\pi}{k^2}|A_{res}|^2$

$\sigma_{sc,0} = \frac{\frac{\pi}{k^2}.4\left(\left(\frac{kR}{a}\right)^2\right)}{(E-E_0)^2 + (\frac{b}{a} + \frac{kR}{a})^2}$

The plot of neutron energy Es with $\sigma_{sc,0}$ is shown in fig.8.1

If we assume that b=0 and k does not vary at the peak, then

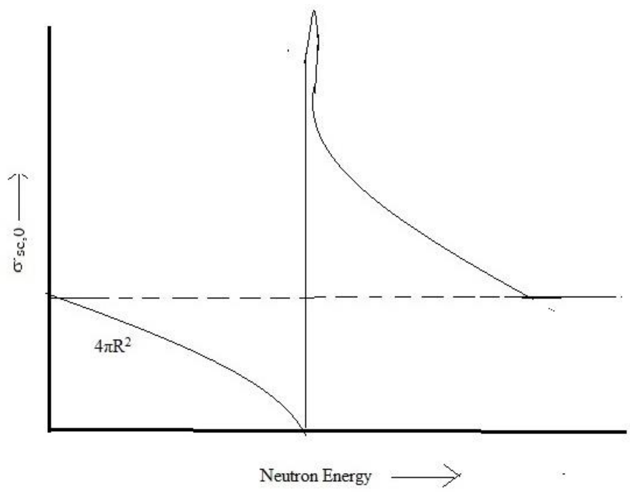


Fig.8.1

$\frac{kR}{a} = \frac{\Gamma_{sc,0}}{2}$

Where $\Gamma_{sc,0}$ is the full width at half maximum, known as resonance width.

Moreover, f_0= -a (E-E_s)-ib at a constant value E_s

$\frac{\partial f}{\partial E}$ = -a

a= $(-\frac{\partial f}{\partial E})_E$

The reaction cross section becomes,

$$\sigma_{r,0} = \frac{\pi}{k^2}\,(1-|\eta_0|^2)$$

$$= \frac{\pi}{k^2}\,[1-|\frac{-a(E-E_S)-ib+ikR}{-a(E-E_{S)-ib-ikR}}|^2]$$

Simplifying we get,

$$\sigma_{\ r,0} = \frac{\pi}{k^2}\frac{\left(\frac{2b}{a}\right)(\frac{2kR}{a})}{(E-E_0)^2+(\frac{b}{a}+\frac{kR}{a})^2} \qquad \text{----------------------(4)}$$

If there is no absorption then b>0 and the resonance peak increases by 2b/a which is known as the reaction width $\Gamma_{r,0}$. Substituting for b/a and kR/a, equ (3) and equ. (4) become,

$$\sigma_{sc,0} = \frac{\pi}{k^2}\frac{\Gamma^2_{sc,0}}{(E-E_0)^2+(\frac{b}{a}+\frac{kR}{a})^2} \qquad \text{--------------------(5)}$$

and $\sigma_{sc,0} = \frac{\pi}{k^2}\frac{\Gamma_{sc,0}+\Gamma_{r,0}}{(E-E_0)^2+\frac{1}{4}(\Gamma_{sc,0}+\Gamma_{r,0})^2}$ ----------------(6)

Equ. (5) and equ (6) are known as Breit-Wigner one level formula. From equ. (6), it is seen that $\sigma_{r,0}$=0 if $\Gamma_{sc,0}$=0. This shows that resonance absorption is always associated with scattering.

If A_{pot} is also into account, then

$$\sigma_{sc,0}= \frac{\pi}{k^2}|\frac{\Gamma_{sc,0}}{(E-E_S)+\frac{i}{2}(\Gamma_{sc,0}+\Gamma_{r,0})} + 2ie^{ikR\ SinkR}|$$

If kR << 1, sin kR=kR and e^{ikR}=1, then

$$\sigma_{sc,0}= \frac{4\pi}{k^2}|\frac{\Gamma}{2(E-E_{s)}+i\Gamma_{sc,0}} + kR|^2$$

For away from resonance when 2 (E-Es) >> $\frac{\Gamma}{kR}$,

$$\sigma_{sc,0}= 4\pi R^2|\frac{\frac{\Gamma}{kR}}{2(E-E_{0)}} + 1|^2 \qquad \text{-------------------------(7)}$$

$\sigma_{sc,0}= 4\pi R^2$ which denotes the hard sphere scattering. For negative values of (E-E_0) in equ.(7), the first term will have a negative real component, partially canceling the value of kR. As resonance is reached at E=Es, the first term contains –I and hence the first term dominates and $\sigma_{\ sc,0}$ decreases.

Inside the nucleus the value vector (α) is related to the energy as,

$$\alpha^2 = \frac{2m(E+V_0)}{\hbar^2}$$

Considering l=0 neutrons, resonance occurs when f_0=0

f_0 =-αRω+αR=0

i.e., αR= (n+$\frac{1}{2}$)π where n=0,1,2,3,….

The distance between resonance peaks,

$$\Delta\alpha = \frac{\left(n+\frac{3}{2}\pi\right)-(n+\frac{1}{2}\pi)}{R} = \frac{\pi}{R}$$

We know that, $\alpha^2\hbar^2$= 2m (E+V_0)

Differentiating with respect to α,

$$2\alpha\hbar^2\Delta\alpha = 2m\Delta E$$

$$\Delta E = \frac{\hbar^2\alpha\Delta\alpha}{m} = (\frac{\hbar\alpha}{m})\,(\hbar.\Delta\alpha)$$

$$\Delta E = \frac{\hbar\alpha}{m}(\frac{\pi}{R}).\hbar$$

But, $\frac{\hbar\alpha}{m}$ = v = velocity

$= \frac{2R}{t_0}$ where t_0 is the transit time of the particle inside the nucleus.

Then,

$$\Delta E = v(\frac{2\pi\hbar}{vt_0})$$

$$\hbar = \frac{\Delta E}{2\pi}\,t_0$$

$= \frac{D}{2\pi}\,t_0$ where ΔE=D is the resonance width.

By the principle of uncertainty,

$$\Delta E.\Delta t=\hbar$$

$$\Gamma\tau t_0=\hbar$$

$$\Gamma= \frac{\hbar}{\tau_0} =\frac{D}{2\pi}.\frac{t_0}{\tau}$$

But $\frac{t_0}{\tau}$ is defined as the barrier penetration factor.

i.e., $p=\frac{t_0}{\tau} =\frac{4k\alpha}{(k+a)^2}$

For slow neutrons, k<<<<a.

$$P=\frac{4k\alpha}{(\alpha)^2} =\frac{4k}{\alpha}$$

$$= 4\sqrt{\frac{E}{V_0}}$$

$$\Gamma = \frac{D}{2\pi} 4\sqrt{\frac{E}{V_0}}$$

$$= \frac{2D}{\pi}\sqrt{\frac{E}{V_0}} \text{----------------------------------} (8)$$

Equ (8) gives the level width of a virtual level formed by a neutron in a square well potential.

8.7 LET US SUM UP

- The cross section of a nuclear reaction is defined as the effective area offered by the nucleus for the incoming particle for a particular type of process of take place.
- The incident particle approaches a target nucleus, it gets absorbed by the target nucleus. This forms a compound nucleus in the excited state.
- The interaction of neutrons with nuclei depends on the energy of the incident particle.

8.8 Review questions:

1. Obtain Breit-Wigner one level formula for resonace scattering. Deduce the level width.
2. Give the theory of a compound nucleus formation and to its decay.

8.9 Further reading:

1. Nuclear Physics D.C.Tayal, Himalaya house, Bombay
2. Nuclear Physics V.Devanantham, Narosa Pub., New Delhi

UNIT IX NEUTRONS

Structure:

9.1 Introduction
9.2 Objectives
9.3 Interaction of neutrons with matter
9.4 Thermal neutrons
9.5 Neutron cycle in a nuclear reactor: Four factor formula
9.6 critical size
9.7 Let us sum up
9.8 summary
9.9 Review of questions
9.10 Further readings

9.1 Introduction

The neutron is a subatomic particle, symbol n or n_0, with no net electric charge and a mass slightly greater than that of a proton. Protons and neutrons constitute the nuclei of atoms. Since protons and neutrons perform similarly within the nucleus, and each has a mass of around one atomic mass unit, they are both denoted to as nucleons. Within the nucleus, protons and neutrons are bound together through the nuclear force. Neutrons are required for the stability of nuclei, with the exception of the single-proton hydrogen atom. Neutrons are produced abundantly in nuclear fission and fusion. They are a primary contributor to the nucleo synthesis of chemical elements within stars through fission, fusion, and neutron capture processes.

9.2 Objectives: Interaction of neutron with matter is discussed- Thermal neutrons are deliberated- neutron cycle in a thermo nuclear reactor is discussed- Critical size is defined.

9.3 Interaction of neutrons with matter:

The interaction between neutrons and matter in bulk is very different from that of charged particles of γ-rays and is a subject that requires special treatment. It was found by Fermi (1934) that the radioactivity induced in targets bombarded with neutrons is increased when the neutrons are made to pass through a hydrogenous material placed in front of the target. The neutrons are slowed down in the hydrogenous material, apparently without being absorbed, and the slower neutrons have a greater probability of inducing radioactivity than do more energetic neutrons. The conversion of fast neutrons into slow neutrons has been investigated in great detail both experimentally and theoretically, and the importance of slow neutrons and the slowing down process has been demonstrated beyond question.

The amount of energy that a neutron loses in a single collision can be calculated by solving the equations of conservation of energy and momentum for the energy of the neutron after the collision. But there is another method which is simpler and neater, and also illustrates some ideas which are very useful in the treatment of nuclear collisions. This method involves the use of two reference system: the first is laboratory or L-system, in which the target nucleus is assumed to be rest before the collision, and is approached by the incident neutron; the second is the centre of mass or C-system, in which the centre of mass of neutron and nucleus is considered to be at rest and both the neutron and the nucleus approach it. The collision process can be described from the view point of an observe moving with the center of mass, and it turns out that the equations needed for the description are relatively simple. They can be solved quite easily and the results can then be transformed back to the L-system. In view of the importance of the slowing-down process and of the usefulness of the center of mass reference system in nuclear physics, some of the less complex aspects of the process will be treated in details.

The relationship between the L-and C-systems is shown in Fig.9.1. In the L-system, before the collision, the neutrons on mass m moves towards the nucleus with speed v_0, momentum mv_0 and energy E_0; the nucleus of mass M is assumed to be at rest. The speed of the centre of mass V_c is given by

$$V_c = v_0 \frac{m}{M+m} \qquad \text{----------------------------------}(1)$$

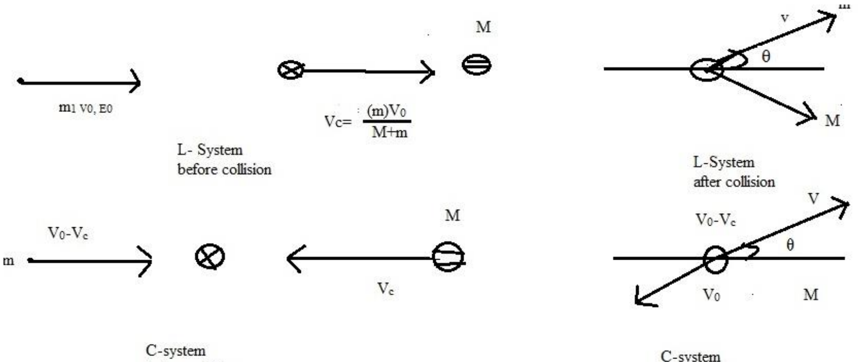


Fig. 9.1

After the collision, the neutron moves with speed v and the energy E, at an angle θ with its original direction, and the nucleus moves off at some angle with the original direction of the neutron.

In the C-system, before the collsion, the neutron moves to the right with speed.

$$V_0 - V_c = v_0 \frac{m}{M+m} \text{--------------------------}(2)$$

and the nucleus moves to the left with speed V_c, the total momentum, as measured in the C-system is,

$$m\left(\frac{Mv_0}{M+m}\right) - M\left(\frac{mv_0}{M+m}\right) = 0$$

Since momentum is a vector quantity and the velocity of the nucleus is opposite in direction to that of the neutron. After the collision, the neutron moves at an angle φ with its initial direction. Since the total momentum must be conserved, its value must be zero after the collision, and the nucleus must move off at an angle (180°+φ) with the direction of the incident neutron. The fact that the momentum is zero in the C-system before and after the collision makes the arithmetic in this system simpler than in the L-system. The observer in the C-system sees only a change in the direction of the neutron and the nucleus as a result of the collision, and the two particles in the C-system must be the same as they were before the collision; otherwise there would be a change in the total kinetic energy of the two particles. The total effect in the C-system is, therefore, to change the direction of the velocities but not their magnitudes. In the L-system, in which the nucleus was originally at rest, the magnitude of the velocities are changed, and the directions are not opposite. The neutron, which is scattered through an angle θ, has a velocity v which is the vector sum of the velocity of the neutron in the C-system and the velocity of the center of mass. The relation between the different velocities is shown in the vector diagram of fig.9.2

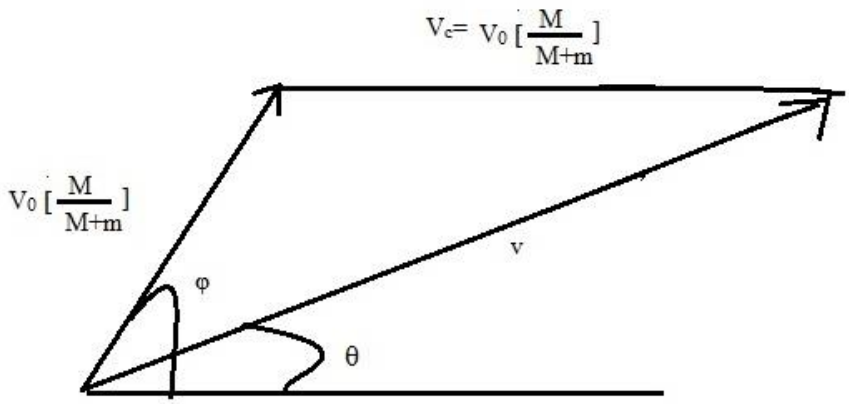


Fig.9.2

There are two cases of special interest for which the speed of neutron after the collision is readily obtained from the figure.

In a glancing collision φ=0 and

$$v = v_0\frac{M}{M+m} + v_0\frac{m}{M+m} = v_0$$

The amount of energy lost by the neutron is negligible, and $E=E_0$. In a head-on collision, φ=180°, and the speed of the neutron is, v = $v_0 \frac{M}{M+m}$-$v_0\frac{m}{M+m}$= $v_0(\frac{M-m}{M+m})$

$\frac{E_{min}}{E_0} = \frac{1/2mv^2}{\frac{1}{2}mv_0^2}$= $(\frac{M-m}{M+m})^2$ ------------------------------(3)

The neutron loses the most energy in a head-on collision; when the moderator is graphite, M=12, m=1 and

$\frac{E_{min}}{E_0}$ =$(\frac{12-1}{12+1})^2$ = 0.72

A neutron can therefore lose up to 28 % of its energy in a collision with a carbon nucleus; 1 MeV neutron can lose as much as 0.28 MeV per collision, and a 1 eV neutron up to 0.28 eV.

For intermediate values of φ, the neutron speed after the collision can be found as a function of φ by applying the trigonometric law of cosines of Fig.9.2

$$v^2=v_0^2(\frac{M}{M+m})^2+ v_0^2(\frac{m}{M+m})^2+ 2v_0^2(\frac{M}{M+m})(\frac{m}{M+m}) \cos \varphi$$

The ratio of the neutron energy E after collision to the initial energy E_0 is then,

$\frac{E}{E_0} = \frac{v^2}{v_0^2} = \frac{M^2+m^2+2Mmcos\varphi}{(M+m)^2}$ --------------------(4)

If the ratio of moderator mass to neutron was M/m is called A, equation (4) becomes,

$\frac{E}{E_0} = \frac{A^2+1+2A\,cos\varphi}{(A+1)^2}$ --------------------------------(5)

The mass ratio A can be taken equal to the mass number of the moderator without introducing any significant error, since m is close to unity and M is very close to an integer. It is convenient to express the energy ratio in terms of the quantity.

r = $(\frac{A-1}{A+1})^2$ -------------------------(6)

which is a measure of the maximum energy that can be lost by the neutron in a single collision. Equation (5) then becomes,

$\frac{E}{E_0} = \frac{1+r}{2} + \frac{1-r}{2}$ cosφ --------------------------------(7)

The greatest energy loss occurs for φ=180°, when cosφ =-1 and E=rE_0; for φ=0, cosφ=1, and E=E_0.

The scattering angle φ in the center of mass system can now be related to the scattering angle in the laboratory system. It is evident from Fig.9.3 that cos φ=D/B, where

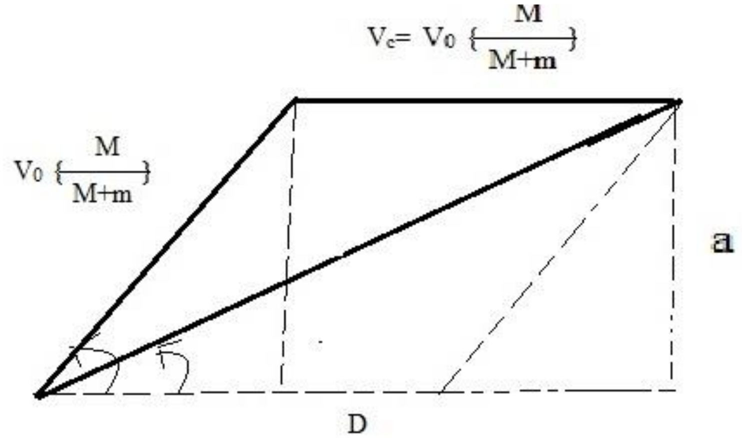


Fig. 9.3

D = $v_0\frac{M}{M+m}\cos\varphi + v_0\frac{m}{M+m}$

B = $v_0\frac{M}{M+m}\sin\varphi$

So that, cot φ= $\frac{cot\varphi+1/A}{sin\varphi}$ --------------------------------(8)

The cosine of an angle can be obtained from the cotangent by means of the relation.

cos θ=$\frac{cot\varphi}{(1+cot^2\theta)^2}$ --(9)

The quantity needed is the average value of cosθ, which can be obtained by integrating equation (9) over the possible values of φ, the scattering angle in the C-system. This integration depends on the probability that a neutron will be scattered through an angle between φ and $\varphi + d\varphi$, but this probability is not known a priori. Both experimental results and a rigorous theoretical treatment of collision process show that the scattering is spherically symmetric in the C-system, provided that the initial energy of the neutrons is less than 10 MeV. This condition is satisfied in most cases of interest, in particular for the neutrons resulting from nuclear fission. Equation (9) can then be integrated over the element of solid angle 2πsinφ dφ, and

$$\overline{\cos\theta} = \frac{1}{4\pi}\int_0^{\pi} cos\theta.2\pi.sin\varphi.d\varphi$$

$$=\frac{1}{2}\int_0^{\pi}\frac{1+A\,cos\varphi}{(1+A^2+2Acos\varphi)^2}\sin\varphi\ d\varphi$$

If cos φ is set equal to x,

$\overline{\cos\theta} = \frac{1}{2}\int_{-1}^{+1}\frac{1+Ax}{(1+A^2+2Ax)^2}dx = \frac{2}{3A}$ ---------------------------(10)

When A is large i.e., for heavy scattering nuclei, $\overline{\cos\theta}$ is small and the scattering in the L-system is practically isotropic. Neutrons which collide with heavy nuclei are scattered forward as often as they are scattered backward. When A is small, as for light nuclei, more neutrons are scattered forward than backward.

The average energy loss per collision can now be calculated; the calculation involves a more useful quantity, the average decrease for collision in the logarithm of the neutron energy, denoted by ε. The quantity is a convenient one to use in neutron slowing-down calculation because it is independent of the neutron energy. Since E/ E_0 , as given be equation (7), is a linear function of $\cos\varphi$, and all values of $\cos\varphi$ are equally probable, it follows that all values of E/E_0 are equally probable. The probability PdE that a neutron of initial energy E_0 will have an energy, after one collision, between E and E+dE is given by,

$$\mathrm{PdE}=\frac{dE}{E_0(1-r)} \qquad \text{---------------------------------(11)}$$

Where E_0 (1-r) represents the entire range of energy values which a neutron can have after one collision. By definition,

$$\varepsilon=\overline{\ln Eo-\ln E}=\overline{\ln\frac{Eo}{E}}$$

Then,

$$\varepsilon=\int_{-Eo}^{Eo} ln\frac{E_0}{E}\,\mathrm{PdE}=\int(ln\frac{E_0}{E})\,\frac{dE}{E_0(1-r)}$$

If x is set equal to E/E_0,

$$\varepsilon=\frac{1}{1-r}\int \ln x.dx$$

Or

$$\varepsilon=1+\frac{1}{1-r}lnr$$

$$=1+\frac{[\frac{A-1}{A+1}]^2\ln[\frac{A-1}{A+1}]^2}{1-[\frac{A-1}{A+1}]^2} \qquad \text{------------------(12a)}$$

And ε is independent of energy, as stated above.

Equation (12a) can be rewritten in the form

$$\varepsilon=1-\frac{(A-1)^2}{2A}\ln(\frac{A+1}{A-1}) \qquad \text{---------------------------(12b)}$$

For A>10, a convenient approximation for ε, good to about one percent, may be used.

$\varepsilon = \frac{2}{A+\frac{2}{3}}$ ----------------------------------(13)

The formula for ε, Equ (12a) and (12b), break down for two special cases, A=1 (hydrogen) and A=∞, because the functions on the right sides of the equations are no longer determinate. By taking the appropriate limits as A→1, and as A→ ∞, value of ε can be determined. For the important case A=1, ε=1 and the average value of ln ($\frac{E_0}{E}$) is unity. For A→ ∞, ε →0, so that a neutron loses practically no energy in an elastic collision with a heavy nucleus. These results can also be obtained from equation (6) which allows that r→1 and Equ. (7) which shows that E/Eo →1. In fact, for large values of A, r can be expand in the series

$$r = 1-\frac{4}{A}+\frac{8}{A^2}-\frac{12}{A^3}+\ldots\ldots\ldots\ldots$$

and for values of A greater than 50, r =1-4/A. Remembering that the maximum energy loss occurs for cosφ =-1, we get

$$E = rE_0 = E_0\ (1-4/A)$$

It is sometimes useful to consider the average energy after a collision

$$\overline{E} = \int_{-Eo}^{Eo} EP(E)dE = \frac{E_0(1+r)}{2} \text{ --------------------------(14)}$$

For hydrogen (A=1), r=0 and the average of a neutron after a collision with a proton is just half of the initial energy. In a head –on collision between a neutron and a proton, which have very nearly equal masses, the former can lose all the energy; this result follows from equ (7), since cosφ =-1 and r=0. This possibility disintegration hydrogen from other moderators. For a collision with a carbon atom, r=0.72, and $\overline{E}$ =0.86 E_0. For A=200, r=0.98, $\overline{E}$ =0.99E_0 and E_{min}=0.98E_0.

When ε is known, the average number of collision needed to bring about a given decrease in neutron energy can easily be calculated.

9.4 Thermal neutrons:

During the slowing down process, neutrons reach a state in which they are in thermal equilibrium with the atoms of the moderator. In each collision neutrons may lose small amount of energy. This results in a state where the energy of the neutrons becomes the thermal energy of the atoms of the moderator. These neutrons are known as thermal neutrons. Thermal neutrons obey Maxwell law of distribution of velocities given as,

$$n(v)dv=4\pi.n\left(\frac{m}{2\pi kT}\right)^{3/2} v^2\ e^{mv^2/2kT}.dv \text{ -------------------(1)}$$

Where n is the number of neutrons per unit volume, n(v) dv is the number of neutrons with velocities between v and v+dv, k is Boltzmann's constant, and T is the temperature in Kelvin. From equ (1) it is clear that,

n(v) = 0 for v=0 and also for v=∞

Hence, n(v) will have a maximum for some value of v, known as most probable velocity (v_0). Differentiating equ. (1) with respect to 'v' and equating it to zero we get,

$$v_0 = \left(\frac{2kT}{m}\right)^{1/2}$$

The energy corresponding to the most probable velocity is E0 given as,

$$E_0 = 1/2\, m v_0^2 = kT$$

The energy distribution of the neutron is,

$$n(E)dE = \frac{2\pi . n}{(nkT)^{3/2}} e^{-E/kT} E^{1/2} dE \quad \text{-----------------(2)}$$

9.5 Neutron cycle in a nuclear reactor: Four factor formula

Consider the fission of a U^{231} nucleus by a thermal neutron. In this process. Let γ neutrons be emitted. These neutrons are the fact neutrons having an average energy greater than the threshold energy of the nucleus. Some of them can produce further fission. Let the total number of fast neutrons be raised to γε where ε is found to be equal to 1.03. A fraction If of these fast neutrons escape out where If is known as leakage factor for fast neutrons. The number of neutrons reaching the moderator becomes γε(1-I_f) and they are slowed down. The slowing down process is due to the collision with the atoms of the moderator. During the process of slowing down process, some of the neutrons are captured by U^{238} to become U^{239}, which decays into Np ^{239} and then Pl^{239}. Let a fraction γε(1-I_f)p neutrons escape from the resonance absorption. The factor p is known as resonance escape probability. Let a fraction I of the slow neutrons escape out. The remaining slow neutrons is γε(1-I_f) p (1-I_f). Of these neutrons, let a fraction f be absorbed by Uranium. The remaining γε(1-I_f)p(1-I_f) (1-f) neutrons are absorbed by materials other than Uranium. The number of slow neutrons available for the chain reaction becomes γε(1-I_f)p(1-I_{tr}) f. The quantity f is known as thermal utilization factor. All the neutrons absorbed by Uranium cannot cause fission. Some of the neutrons are absorbed by U^{238} to become U^{239}. Let a fraction 'g' of the neutrons are absorbed by U ^{235} to become U^{236}. The number of neutrons that can cause fission becomes γε(1-I_f) p (1-I_{tr})f_g. Let the fraction be g=$\frac{\sigma_f(U)}{\sigma_a(U)}$. The number of second generation fissions in U^{235} per fission of U235 is known as reproduction factor (k). Then,

$$k = \gamma\varepsilon(1-I_f)\, p(1-I_{tr})\, f \frac{\sigma_f(U)}{\sigma_a(U)} \quad \text{-------------------------------(1)}$$

Here $\gamma \frac{\sigma_f(U)}{\sigma_a(U)}$ is the number of fast fission neutrons produced per thermal neutron absorbed in Uranium. Let it be η.

$$k = \eta\varepsilon\left(1 - I_f\right)p(1 - I_l)\, f \quad \text{---------------------------------(2)}$$

where k denotes the multiplication factor. For the chain reaction to continue, k must be unity. If k is less than unity, the chain reaction cannot take place. The system is said to be sub critical. If k >1, the reaction becomes violent, then the system is said to be super critical. Critical size of a nuclear reactor is the size of the reactor for which k =1. If there is no leakage of neutron then $I_f=0$, $I_l=0$. Then,

$$k = \eta\varepsilon.pf \quad \text{--(3)}$$

This relation is known as four factor formula. If the reactor contains only U^{235}, then ε and p are unity, so that,

$$k = \eta f$$

where η depends on the nuclear properties of the fuel. But p and f depend on nuclear properties of the fuel, moderator and other materials present inside the reactor.

Calculation of k forms an important point in the design of a nuclear reactor. The necessary condition for the reaction to proceed is k>1. When this condition is satisfied, the neutron density grows which is given as,

$$\frac{dn}{dt} = \frac{n(k-1)}{\tau} \quad \text{---------------------------(4)}$$

Where τ is the generation time,

The solution of Equ (4) is,

$$n(t) = n(0).\, e^{(k-1)t/\tau}$$

The neutron density is controlled by means of cadmium rods which are good slow neutron absorbers.

9.6 Critical size:

The critical size of a nuclear reactor not only depends on the value of the reproduction factor, but also on the shape and volume of the reactor. Consider a finite homogeneous reactor in the form of slab, with a finite thickness along the x.dx and infinite thickness on y and z directions. Let a source of neutron be placed at any arbitrary origin in the slab. In a steady state of neutron leakage, Rate of neutron leakage+ rate of neutron absorption= rate of neutron production.

Consider an element of thickness dx at any distance 'x' from an arbitrary point.

The rate of leakage = $-\frac{\lambda_{tr}v}{3}\frac{d^2n}{dx^2}$ dx ----(1)

Where λ_{tr} is transport cross section of the reactor material, n is the neutron density, and v is the velocity of the neutron.

Rate of production in the element 'dx'= nv $\sum_a k_\infty$dx

Hence, $-\frac{\lambda_p v d^2 n}{3dx^2}$ +nv$\sum_a$ = nv$\sum_a k_\infty$

$\frac{d^2n}{dx^2}+\frac{3\sum_a(k_\infty-1)n}{\lambda_p}= 0$ --------------------------(2)

Let L be the diffusion length given as

$L^2 = \frac{\lambda_{tr}}{3\sum_a}$

Equ (2) becomes

$\frac{d^2n}{dx^2}+\frac{k_\infty-1}{L^2}$ n= 0 ------------------------------(3)

If the reactor is well moderated then,

$M^2=L^2+L_s^2$, where M is the migration area and Ls is the slowing down length.

Let n(x) = 0 and x=a/2 ---------------------(4)

The solution of equ (3) given the density of neutrons given as,

n(x)=A cos Bx+c.sin Bx where A and C are constants and $B_2= \frac{k_\infty-1}{M^2}$, since sin Bx asymmetrical of x=0, c=0.

x= A cos Bx.

By the condition given by equ.(4)

$\frac{Ba}{2}=\pi/2$

a= $\frac{\pi}{B}=\frac{\pi M}{\sqrt{k_\infty-1}}$ --------------------------(5)

Equ.(5) gives the critical thickness.

The neutron density becomes,

$n(x) = A \cos\frac{\pi x}{a}$

Spherical Reactor:

Let R_c be the critical radius of a spherical reactor given as,

$$R_c = \frac{\pi M}{\sqrt{k_{\infty-1}}}$$

For a critical reactor of size Ac, we have,

$$A_c = \frac{\sqrt{3}\,\pi M}{\sqrt{k_\infty - 1}}$$

For a sphere, the critical volume becomes,

$$V_c = \frac{4}{3}\pi R_c^3 = 130\left(\frac{M^2}{k_\infty - 1}\right)^{1/2} \quad \text{------------------------(6)}$$

For a cubical reactor,

$$V_c = A_c^2 = 161\left(\frac{M^2}{k_\infty - 1}\right)^{1/2} \quad \text{---------------------------(7)}$$

Equ (6) and Equ(7) represent the critical size of spherical and cubical reactor.

9.7 LET US SUM UP

- ❖ The interaction between neutrons and matter in bulk is very different from that of charged particles of γ-rays and is a subject that requires special treatment.
- ❖ During the slowing down process, neutrons reach a state in which they are in thermal equilibrium with the atoms of the moderator.
- ❖ The critical size of a nuclear reactor not only depends on the value of the reproduction factor, but also on the shape and volume of the reactor.

9.8 Review questions:

1. Explain the interaction of γ-rays with matter. Obtain an expression for the energy lost by a neutron per collision.
2. What are thermal neutrons? Explain
3. Explain neutron cycle in a nuclear reactor. Derive four factor formula.
4. What is critical size of a reactor? Derive an expression for the critical size of a nuclear reactor.

9.9 Further reading:

1. Nuclear Physics- Gupta and Roy, Arunabha Sen Publishers, Kolkata

2. Nuclear physics- D.C.Tayal, Himalaya House, Bombay

UNIT X NUCLEAR REACTORS

Structure:

10.1 Introduction

When $_{92}U^{235}$ endures fission after attacked by a neutron, it also releases an extra neutron. This extra neutron is then available for initiating fission of another $_{92}U^{235}$ nucleus. In fact, on an average, 2 neutrons per fission of uranium nucleus are released. The fact that more neutrons are produced in fission than are consumed raises the possibility of a chain reaction with each neutron that is produced triggering another fission.

10.2 Objectives: Types of nuclear reactors are discussed- cylindrical and spherical nuclear reactors are deliberated- sub-nuclear particles (elementary ideas) is discussed- source of stellar energy is discussed- controlled thermo nuclear reactions is deliberated.

10.3 Types of nuclear reactors:

In nuclear fission, we have seen that a heavy nucleus breaks up into two fragments with an energy release of approximately 200 MeV per fission. The power has been connected and is now a reality. A process just opposite to it, is the nuclear fusion in which two lighter nuclei fuse together to form a heavier nucleus. Nuclear fusion is not exactly a new phenomenon. It has been generating the power of the sun and the stars for billions of years which formed one of the most intriguing questions before the discovery of nuclear fusion. As a matter of fact physicists discovered the fusion reaction in laboratory before the discovery of fission. But to create and control fusion power on earth is a problem of totally different order from harnessing fission which as we have seen, is much easier to produce and control (as in nuclear reactors) and that is why fission received more attention than fusion.

The sun radiated energy at the rate of 1026 joules per second. The chemical reactions cannot possibly supply this much amount of energy such a long-time. Another possible source this energy may be conversion of gravitational energy into heat but this also could not give an adequate explanation. The failure of ordinary sources of energy to explain this energy release led to the thinking that the source of energy might be nuclear.

In order that nuclear reactions may act as the sources of stellar energy, the following conditions must be fulfilled.

1. The reaction must be exothermic, i.e the sum of the masses of the products of the nuclear reaction must be less than the masses of the reacting nuclei. The mass decrement will then yield energy.
2. Reactions involved should be those which could take place at such high temperatures and density as at the sun.
3. The reacting nuclei should have sufficient abundance at the sun and the stars where the reaction takes place.

The first condition implies that the nuclei reactions of interest are either the fusion reactions of light nuclei into a heavier nucleus, or nuclear fission or the radioactive decay of heavy nuclei. Since the abundance of heavy nuclei on the sun is very low (about 90 % of sun's weight consists of higher elements like hydrogen and helium with approximately equal proportion of each element), the possibility of a fission reaction and radioactive decay is ruled out. Therefore it seems quite inevitable to accept that it is the fusion reactions which we responsible for the energy release in the sun and the stars. The fusion reactions are very much dependent on temperature and also the rate of fusion decreases rapidly with increasing charges of colliding nuclei.

Fission was discovered in 1939 when Hahn and Strass man discovered the presence of rare-earth elements in uranium after irradiation by neutrons. L. Meitner and O. Frisch then interpreted this production as being due to neutron-induced fission of uranium. This discovery was followed rapidly by applications since, on December 2, 1942; Enrico Fermi at the University of Chicago produced a chain reaction in a system consisting in a periodic stack of natural uranium spheres separated by graphite moderators. Fermi thus demonstrated experimentally the notion of criticality of the size of the stack in order to ensure a chain reaction. This was achieved with a very small total power of the system, $\sim$ 1W. Present power reactors attain powers of $\sim$ 3 GW. The increase in power does not present by any means the same complication as in fusion, as we shall see in the next chapter. Indeed, in the fission process all phenomena are more or less linear, in (great) contrast with controlled fusion systems.

A fission reactor core consists of the following essential elements

Fuel elements, generally consisting of bars containing natural uranium enriched in ^{235}U, or ^{239}Pu. If there is to be a self-sustaining chain reaction, the amount of fuel must be greater than the critical mass defined by geometric losses. A heat extraction system, generally a fluid, e.g. water in thermal-neutron reactors or sodium in fast-neutron reactors. Its role is to limit the temperature of the core and, in power reactors, to transfer the core's thermal energy to electric generators. (Thermal-neutron reactors only) A moderating system to thermalize the neutrons. This is most simply done by bathing the fuel bars in water. In this case, the moderator also serves as the heat transporter. In the following subsections, we will briefly describe three basic types of fission reactors; those based on thermal neutrons, fast neutrons, and proposed schemes where reactors are driven by particle accelerators.

Fast neutron reactors can be breeders that produce more nuclear fuel than they consume, by using an intermediate fertile nucleus such as ^{238}U or ^{232}Th. This is possible because more than two neutrons per fission are produced in fast-neutron ^{239}Pu reactors. One of these neutrons can be used to maintain the chain reaction and the others can create further ^{239}Pu via.

Neutron absorption on ^{238}U. If the probability for this to happen is sufficiently close to unity, the ^{239}Pu destroyed by fission can be replaced by a ^{239}Pu created by radioactive capture on ^{238}U. The final result is that ^{238}U is the effective fuel of the reactor. The following remarks are in order. Consider, for definiteness, a breeder with the fertile nucleus ^{238}U. The fissile nucleus is ^{239}Pu for two reasons.

Firstly, it is produced in neutron absorption by ^{238}U, which leads to a closed cycle, secondly it is produced abundantly in nuclear technologies, whereas one can only rely on the natural resources of ^{235}U. In order for a fertile capture of a neutron to produce an appreciable amount of the fissile ^{239}Pu inside the fuel, the probability for this capture must not be too small compared to the probability that the various nuclei in the medium undergo fission. This probability depends both on the amounts of ^{239}Pu, ^{235}U and ^{238}U, and on the physical design of the fuel elements. It can be calculated in terms of the amount of various nuclides and of the capture and fission cross-sections of, respectively, ^{238}U and the pair ^{239}Pu – ^{235}U.

10.4 cylindrical nuclear reactors:

A typical (Figure10.1) consists of a cylindrical tank of height Hand radius R filled with D_2O (heavy water) called moderator. Cylindrical tubes, called fuel channels are kept axially in a square array. In each fuel channel there are fuel bundles which are stacked one above the other to a height H. A fuel bundle contains a cluster of cylindrical fuel pins of solid natural UO_2. Fission neutrons, coming outward from a fuel channel, collide with the moderator, losing energy, and reach the surrounding fuel channels with low enough energy to cause further fissions (seeFigures10(a–c)). Heat generated from fission in the pin is transmitted to the surrounding coolant fluid flowing along its length inside the fuel channel. This cylinder is kept horizontal (unlike what is shown in Figure b) to avoid pumping the coolant against gravity. The coolant in these type of reactors is again D_2O and is pressurized to avoid boiling. For this reason, such an NR is called Pressurized. Heavy Water Reactor (PHWR). The heat from the coolant is removed in a heat exchanger and is used to produce steam which runs the turbines to produce electricity. Here, we shall study some of the physics behind the (i) design of the fuel pin, (ii) role of a moderator and finally (iii) dimensions of a NR of cylindrical geometry. However, the study can be extended to other types of reactors

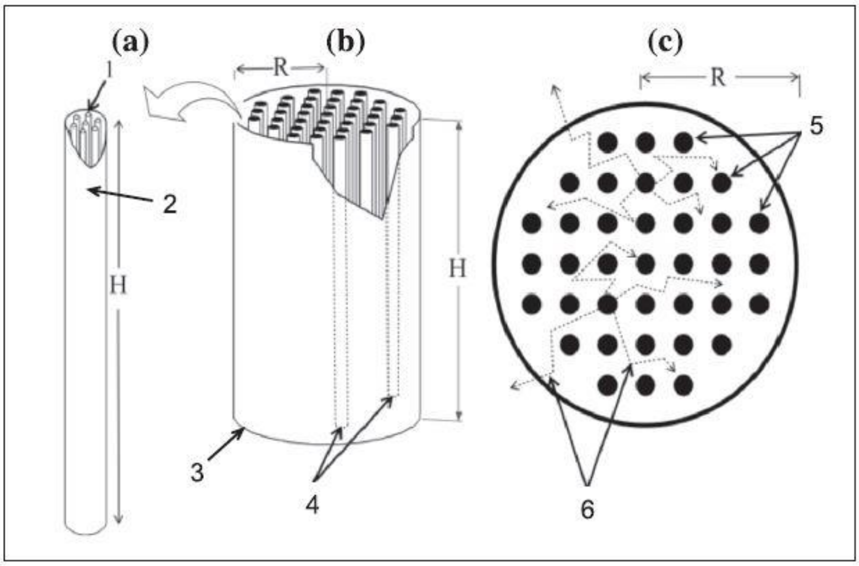
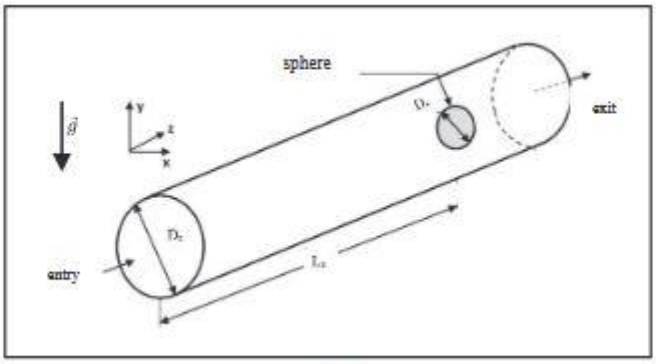


Fig. 10.1 a) Fuel channel b) Fuel channel filled with moderator c) Top view of Fuel channel of cylindrical nuclear reactor.

10.5 spherical nuclear reactors:

PBMRs (Pebble Bed Modular Reactors) and HTR-PM (High Temperature Reactor- Pebble bed Modular) are nuclear reactors that work under high temperature. The difference with other types of nuclear reactors is the geometry of the fuel used. This fuel is spherical in shape. The cooling of the spheres beds could be carried out by inert gases such as carbon dioxide, helium, nitrogen or air.



(a)

Fig. 10.2 Position of the sphere in the spherical reactor.

10.6 Sub nuclear particles:

The nucleus is made up of elementary particles, which are, the protons and the neutrons. The protons and the neutrons contribute to nearly all of the mass of an atom (more than 96%) due to the fact that the mass of an Electron (particles which revolve around the nucleus) is extremely small, and one electron weights 1/1837th that of a proton. A proton is a positively charged particle whereas a Neutron does not have any charge (it is neutral in charge). But there is a reason to why the protons and the neutrons have the charges as +1 and 0 respectively.. A fundamental particle is a particle which cannot be broken down into simpler substances. In fact, the electron is a fundamental particle, whereas, the protons and the neutrons are not, which means that the protons and neutrons can be broken into further smaller particles known as Quarks. Quarks are the fundamental particles which make up a proton and a neutron. There are 6 types of Quarks:

Up Quark

Down Quark

Top Quark

Bottom Quark

Charm Quark

Strange Quark

10.7 Thermo-nuclear Reactions as source of stellar energy:

As we know the sun radiated energy at a very-huge rate (10^{26} joules/sec) and that the possible sources of this energy can be thermo-nuclear fusion reactions. Since hydrogen is by far the most abundant element in the universe, the thermo-nuclear reactions responsible for this huge energy production, most involve hydrogen as reactant. H.A.Bethe in

1932, suggested two sets of thermo nuclear reaction (i) the proton-proton chain reaction and (ii) the carbon-nitrogen cycle.

The proton-proton chain or p-p chain consists of the following reactions:

(a) ${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + \gamma + 0.42$ MeV

Which has a half-life of 7×10^9 yrs

(b) ${}_1H^2 + {}_1H^1 \rightarrow {}_1He^3 + \gamma + 5.49$ MeV

Which has a half-life of 4 sec

(c) ${}_1He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1 + 12.86$ MeV

Which has a half-life of 4×10^5 yrs

As a result of these reactions, four protons are converted into a ${}_2He^4$ nucleus i.e., the effect of the reaction is

$$4\,{}_1H^1 \rightarrow {}_2He^4 + 2e^+ + 2\gamma + 2v$$

The total energy released (Q-value) in the reaction is about 26.7 MeV. The above set of reactions represent one possible route through which four protons are synthesized into a helium nucleus. Holmgren and Johnson is 1958 suggested another possible way in which the p-p chain can take place, which is as follows:

$${}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + e^+ + \gamma$$

$${}_1H^1 + {}_1H^2 \rightarrow {}_2He^3 + \gamma$$

$${}_2He^3 + {}_2He^4 \rightarrow {}_4Be^7 + \gamma \quad] \text{ -----------------------(1)}$$

$${}_4Be^7 + e^- \rightarrow {}_3Li^7 + v + \gamma$$

$${}_3Li7 + {}_1H^1 \rightarrow {}_2He^4 + {}_2He^4$$

$${}_4Be^7 + {}_1H^1 \rightarrow {}_5B^8 + \gamma$$

$${}_5Be^8 \rightarrow {}_4Be^8 + e^+ + \gamma \quad] \text{---------------------------(2)}$$

$${}_4Be^8 \rightarrow {}_2He^4 + {}_2He^4$$

It is believed that the chain (1) is important at comparatively lower temperatures and chain (2) at high temperatures.

10.7.1 Carbon-Nitrogen cycle

Bethe also suggested the possibility of the following reactions, known as the carbon-nitrogen cycle, taking place at the sun and stars

$_6C^{12}+_1H^1 \rightarrow _7N^{13}+\gamma$ Q =1.94MeV, $T_{1/2}$= 10 ^{6} years

$_7N^{14} \rightarrow _6C^{13}+ e^+ + v$ Q = 1.20 MeV,

1.02 MeV, $T_{1/2}$= 10 min

$_6C^{13}+_1H^1 \rightarrow _7N^{14} +\gamma$ Q =7.55 MeV, $T_{1/2}$= 2×10^5 years

$_7N^{14}+_1H^1 \rightarrow _8O^{16}+\gamma$ Q =7.29 MeV, $T_{1/2}$= 2×10^7 years

$_8O^{15} \rightarrow _7N^{15}+e^+ +v$, Q =2.76 MeV, $T_{1/2}$= 2 min

$_7N^{15}+_1H^1 \rightarrow _6C^{12}+_2He^4$, Q=4.96 MeV, $T_{1/2}$= 10^4 years

This also converts four protons into a helium nucleus and the carbon and nitrogen nuclei simply act as catalysts in the nuclear reactions. The total energy released in the process is about the same (26.73 MeV) as in the case of p-p chain.

For quite some time, it was thought that C-N cycle was the main energy source in the sun but now through knowledge of stellar temperature, it is concluded that the p-p chain contributes about 90 % of the nuclear energy. The C-N cycle is a major energy process only at temperatures around 18 million degree centigrade whereas the p-p chain is predominant at lower temperatures. The rate of C-N cycle completions is very much susceptible to temperature, varying as the eighteenth power of temperature around 18 million degrees centigrade. On the other hand the p-p chain completion rate varies roughly as fourth power of temperature. The reaction rates for the two cycles as a function of temperatures are shown in Fig.10.3

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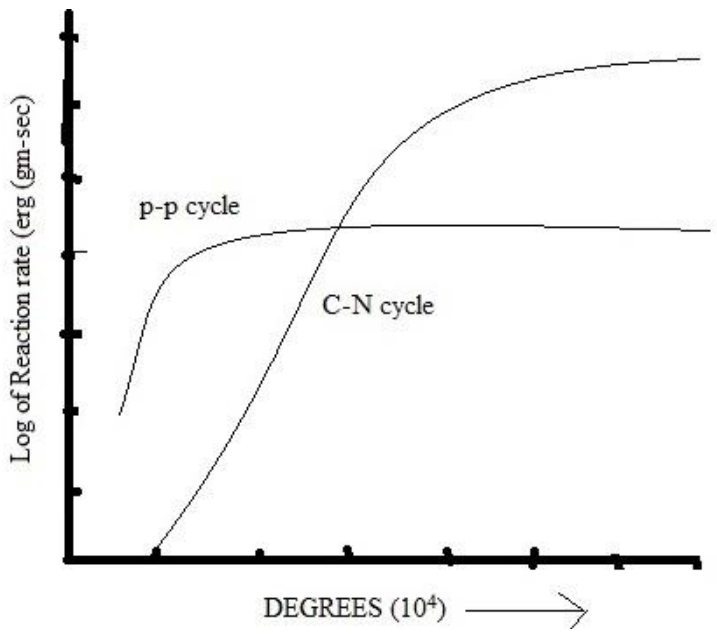

Fig. 10.3

10.8 Controlled thermo nuclear reactions:

The fusion reaction is a device within which the nuclear fusion reactions could take place in a controller manner and useful quantities of energy in excess of that required to operate the device could be extracted from it. We have studied that nuclear fusion reactions release a large amount of energy which is responsible for the energy radiation by the sun and other stars at such a huge rate. But these reactions take place in an uncontrolled manner.

We know that for a fusion reaction to take place, the colliding particles should have enough energies to overcome the Colombian repulsive forces between them. This needs energies of the order of 0.1 MeV which corresponds to a temperature of approximately 10^8 K. This means that for a fusion reaction to take place temperatures of this order should prevail initially. At such high temperatures required for fusion, the atoms are completely stripped of their electrons. i.e., the atoms are completely ionized and the free electrons move about rapidly. The mixture of electrons and ionized atoms is still neutral. This completely ionized and electrically neutral state of matter is called plasma. The matter contained in stars, sun and galaxies is in this plasma state and is held there through gravitational force, but the problems of production and confinement of plasma in laboratory are really formidable ones.

The possible fusion reactions that could take place in this plasma are the following

$_1H^1+_1H^2\rightarrow {}_1H^1+_1H^3+$ 4.03 MeV ------------------------(a)

$_1H^2+_1H^2\rightarrow {}_2He^3+_0n^1+$ 2.27 MeV -----------------------(b)

$_1H^2+_1H^3\rightarrow {}_2He^4+_0n^1+$17.50 MeV ------------------------(c)

$_1H^2+_2He^3\rightarrow {}_2He^4+_1H^1+$18.3 MeV -----------------------(d) -------------(4)

$_1Li^6+_1H^1\rightarrow {}_2He^4+{}_2He^4+$22.4 MeV ---------------------- (e)

$_3Li^7+_1H^1\rightarrow {}_2He^4+{}_2He^4+$ 17.3 MeV ---------------------(f)

Of which the reaction taking place in hypdrogen plasma are the only first four viz, (a), (b), (c) and (d). The reaction cross-sections for the above reactions have been studied carefully and vary between 10^{-4} barn and 1 barn as shown in Fig.10.2.

An important parameter for the fusion reaction is the reaction rate which can be represented as,

$R_{12}= n_1n_1< \sigma\ v_{12}>$ reactions/m^2 /sec.

Where n_1, n_2 represent particle densities of the colliding particles, v_{12}, the relative velocity of the particles and $< \sigma\ v_{12}>$ represent the average of the products of cross-section σ and the relative velocity of the particles. If the colliding particles are identical, we have

$R_{11}=1/2\ n_2 < \sigma v>$ reactions/m^2/sec.

As already remarked for thermo-nuclear reaction to be a source of energy for constructive purposes, we need a device called a fusion reactor in which the process could take place under control conditions.

For the operation of a fusion reactor, the first requirement is to attain temperatures of the order 108 K, to convert hydrogen to plasma state. This can of course be done.

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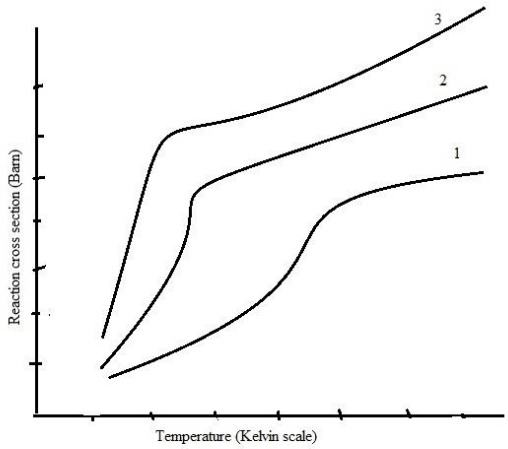

Fig. 10.2

When plasma is produced, the most arduous problem we are confronted with is its confinement since no wall of ordinary material could withstand such high temperatures as it will evaporate immediately.

10.9 Summary:

- The nucleus is made up of elementary particles, which are, the protons and the neutrons.
- The thermo-nuclear reactions responsible for this huge energy production, most involve hydrogen as reactant.
- The p-p chain completion rate varies roughly as fourth power of temperature.
- The fusion reaction is a device within which the nuclear fusion reactions could take place in a controller manner and useful quantities of energy in excess.

10.10 Review questions:

1. What you mean by thermo nuclear reaction? Explain with example.
2. Write a detail note on sources of stellar energy.

3. What are the various types of nuclear reactors? Explain their construction and working.
4. Discuss about the cylindrical nuclear reactor.
5. Write a note on spherical nuclear reactor

10.11 Further reading:

1. Nuclear and particle physics –An introduction – Brian R.Martin
2. Physics of the Nucleus – Gupta and Roy.
3. Joesph A Nathen and Vijay A Singh, ‘The design of nuclear reactor’ Resonance, (2016),843-856.
4. E.M.Medouri et.al., Fuel for high temperature nuclear reactors; thermal hydraulic analysis, AIP Proceedings, 1994.

BLOCK IV ELEMENTARY PARTICLES

UNIT XI FUNDAMENTAL INTERACTIONS IN NATURE

Structure:

11.1 Introduction:

Elementary particles are rather large in number. But only few of them-proton, electron, positron, neutrino and photon-are stable. The rest are all unstable. Some of them have mean lives much larger than the characteristic nuclear time, being defined as the time taken by a pion to travel past a proton ($\sim 4\times 10^{-24}$ s) and decay by weak interaction; few like π^0, $\sum^0$ decay by electromagnetic interaction. A large number again decays by strong interaction, with mean lives $\sim$ characteristic nuclear time.

11.2 Objectives: Classification of fundamental forces are discussed- Particle directory and quantum numbers (charge, spin, parity, iso-spin, strangeness etc) are deliberated.

11.3 Classification of fundamental forces:

To understand the significance of the existence of these large numbers of particles, a study of the fundamental interactions that act between them is worth-making. In nature, there are four different types of fundamental interactions. These are: gravitational, electromagnetic, weak and strong interactions. According to the quantum field theory, all the interactions rely on the mechanism of exchange of quanta. All interactions are transmitted from one particle to the other by successive proves of emission, propagation and absorption of such mediators.

1. Gravitational interaction- It is the weakest of all the fundamental interactions and acts between all bodies having mass and is described by the long range inverse square type Newtonian law of gravitation: $F= Gm_1m_2/R^2$ where m_1, m_2 are the two masses separated by R and G (=6.7 $\times$ 10^{-11} SI units) is the gravitational constant. Subsequently, Einstein extended it to describe gravitational interactions in terms of curvature space. This interaction is believed to be mediated through the quantum of interaction-graviton-which is yet to be discovered and provides a large attractive force between the planets producing the acceleration due to gravity in their vicinity. It is of extreme importance for astral bodies in galaxies and on cosmological scale since large masses and distances is involved.

The gravitational interaction however becomes inappreciable when the interaction of elementary particles, nuclei and atoms are considered and is totally left out.

2. Electromagnetic interactions- This interaction is much stronger than gravitational one and is described by long range inverse square type law: Coulomb's law. It is due to the charges of particles and their motion. In quantum field theory, it is visualized as an exchange of virtual photons which are the quanta of the field. This type of interactions occur in the chemical behavior of atoms and molecules, Rutherford's scattering etc.

3. Weak interactions- The third fundamental interaction, weak, nuclear interaction, takes place in the nuclear characteristic time $\sim 10^{-6}$ to 10^{-10} s. The β-decay of radioactive nuclei and weak decays of strange particles are typical of weak interactions. Unlike the previous two interactions, weak interaction is a very short range force and is mediated through bosons named $W^{\pm}$, Z^0 discovered rather recently by Arnison et al. The intrinsic strength of weak interaction is 10^{-10} times that of electromagnetic interaction.

In weak interactions, the parity is not conserved and this violation of parity distinguishes weak interaction from other fundamental interactions. The important development in the study of this interaction is the unification of weak and electromagnetic interaction into the electro-weak interaction.

4. Strong interaction- This is the strongest force in nature and occurs between a neutron and a proton, or a neutron and a neutron, or a proton and a proton. This is a short range ($\sim 10^{-14}$ -10^{-15}m) attractive force, charge-independent and is mediated through the exchange of π-mesons (π^+, π^-, π^0) which are the quantum of interaction field. It has got a relative magnitude of strength of about 10^{13} times that of weak interactions involving time of interaction $\sim 10^{-23}$ s. It explains strong nuclear interactions between hadrons.

Since protons, neutrons, π-mesons are thought to be built up of more fundamental, entities-quarks-the strong interaction is believed to be mediated through the exchange of another particle called gluons (massless quanta of spin $\hbar$) between the quarks. Neither gluons nor quarks have been observed in free state.

The principle of electromagnetic force between the protons is strong coulomb repulsion, which tends to tear the nucleus apart. (Fig.11.1.b). The gravitational force is an attractive one between every pair of nucleons, but it is smaller by a factor of 10^{39} than the electrical force between the two protons. Thus, the only two forces we have previously uncounted can't account for the existence of nuclei. The only explanation is to recognize that there is a third force in nature, known as nuclear force. The force is very strong at distances of the order of nuclear size, there must be compensate the coulomb repulsion between the two protons. Molecular structure can be accurately accounted for by the e.m. force alone so we may conclude that a distance of the order of the spacing between the nuclei in molecules($=10^{-10}$m) the nuclear force must be negligible, it is a short range force, falling off more rapidly with the distance than $1/r^2$

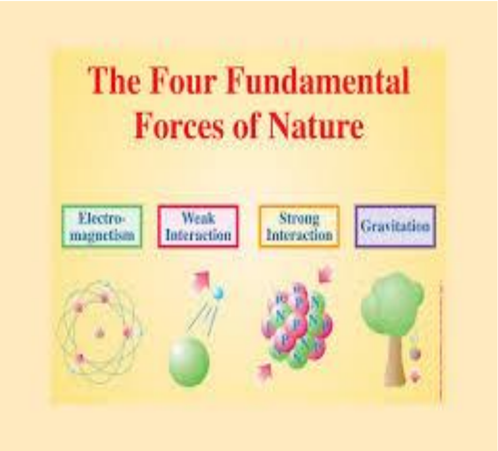

Fig. 11.1.a

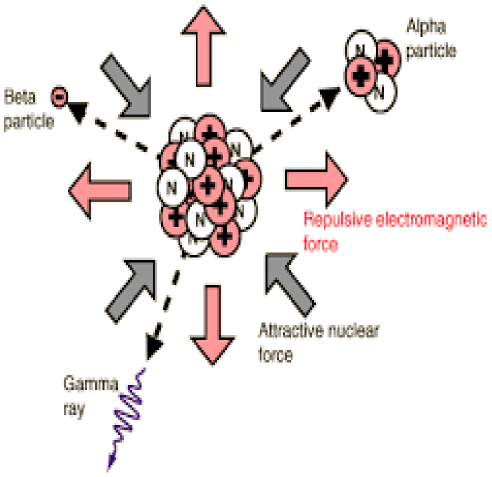

Fig.11.1.b

Short range:

The nucleus would pull in additional protons and neutrons. But this range it must be stronger than electric field otherwise the nucleus can never be stable, here short range means that nuclear force is appreciable only when the interacting particles are very close, at a separation of the order of 10^{-15}m or less. At greater distances, the nuclear forces are negligible. We infer that the nuclear force is of short range because at distances greater than10^{-14}m, corresponding to nuclear dimension, the interaction regulating the scattering of the nucleus and grouping of atoms and molecules is electromagnetic.

Independent of electric current

Neutron and proton must be bound, this means that the nuclear interaction between the proton and two proton and one proton and neutron are basically same. From the analysis of p-p and n-p scattering, scientists have concluded that the nuclear part is essentially the same in both cases. Also the facts that

A) Light nuclei are composed of equal no of p and n.

B) B.E/nucleon is approximately constant.

C) Mass difference of mirror nuclei can be accounted for by the difference in coulomb energy alone, indicative that the nuclear interaction is charge dependent. Because of this property, P and N are considered equivalent insofar.

Relative orientation of the spin of the interacting nucleons

Scattering experiments and by analysis of the nuclear energy levels. It has been found that the energy of a two nucleon system in which the two nucleons have their spins parallel is different from the energy of such a system in which one has spin up and other down. Infact n-p system has bound state, the deuteron in which two nucleons have their spins parallel. (S=1), but no such bound state seems to exist if the spins are anti-parallel. (S=0)

Nuclear force is not in central

It depends on the orientation of the spins relative to the line joining the two nucleons, scientists have concluded this by nothing that even in the simplest nucleon (deuteron), the orbital angular momentum (l) of the two nucleons relative to their center of mass is not constant, contrary to the situation when the forces are central. Therefore, to explain the properties of GND state of the deuteron, such as magnetic dipole and electric quadrupole moment.

We must use a linear combination of s (l=0) and (l=2) wave function. Part of the nuclear force is strongly spin-orbit interaction. Another part tensor force closely resembles the interaction between the two dipoles.

Repulsive core

This means that at very short distances, much smaller than range, the nuclear force becomes repulsive. This assumption has been introduced to explain to constant average separation of nucleons, resulting in a nuclear volume is proportional to the total no. of nucleons, as well as to account for certain features of n-n scattering. Inspite of, all the information about nuclear force, the correct expression for the P.E for nuclear interaction between two nucleons is not yet well known.

One is Yukawa potential, $E_p(r) = -E_0 r_0\, e^{-r/r0}/r$ which is shown in Fig 11.2

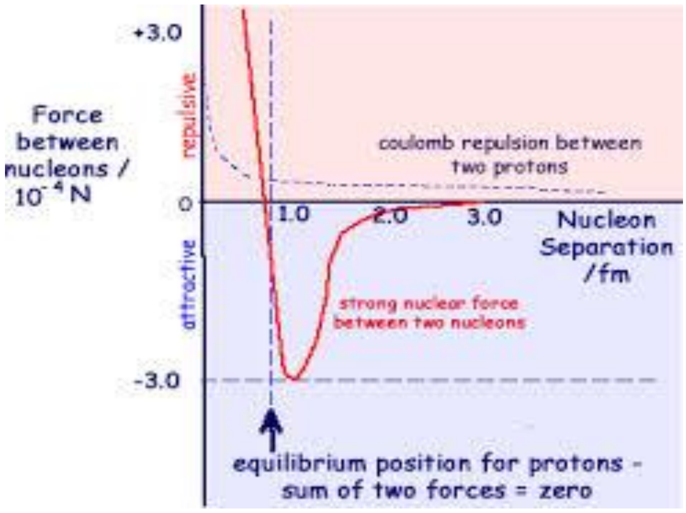

Fig. 11.2

11.4 Particle directory:

Electron

It is the first fundamental particle, which was discovered by Thomson in 1897. It revolves around the nucleus of an atom is different orbits. Electron plays an important role in explaining the physical and chemical properties of substances. Its charge is $-\ 1.6 \times 10^{-19}$ Coulomb and mass is 9.1×10^{-31} kg. Its symbol is e^- (or $_{-1}\beta^0$)

Proton:

It was discovered by Rutherford in 1919 in artificial nuclear disintegration. It has a positive charge (1.6×10^{-19} Coulomb) equal to the electronic charge and its mass is (1.67×10^{-27}kg) 1836 times the electronic mass. In free state, the proton is a stable particle. Its symbol is p+. It is also written as $_1H^1$.

Neutron:

It was discovered by Chadwick in 1932. It carries no charge. Its mass is 1839 times the electronic mass (1.675×10^{-27} kg). In free state the neutron is unstable (its mean life is about 17 minutes), but it constitutes a stable nucleus along with proton. Its symbol is n is $_0n^1$.

Positron:

It was also discovered in 1932 by Anderson. Its charge and mass same as those of electrons, the only difference being that it is positively-charged whereas the electron is negatively charged. Its symbols is e^+ (or $_{+1}\beta^0$)

Antiproton:

It was discovered in 1955. Its charge and mass are same as those of proton, the only different being that it is negatively charge. Its symbol is p^-.

Antineutron:

It was discovered in 1956. It has no charge and its mass is equal to the mass of neutron. The only difference between neutron and antineutron is that if the spin in the same direction, their magnetic momenta will be in opposite directions. The symbol of antineutron is $\overline{n}$

Neutrino and antineutrino:

The existence of these particles was predicted in 1930 by Pauli while explaining the emission of β –particles from radio-active nuclei, but they were observed experimentally in 1956. Their rest mass and charge are both zero but they have energy and momentum. Both neutrino and anti-neutrino are stable particles. The only difference between them is that their spins are in opposite directions. Their symbols are γ and $\overline{\gamma}$ respectively.

Pi-mesons:

The existence of these particles was predicted by Yukawa in 1935 as originator of exchange-forces between the nucleons, but they were actually discovered in 1947 in cosmic rays. Pi-mesons are of three types.

1. Positive pi-meson: It is a positively charge particles whose charge is equal to the electron charge and whose mass is 247 times the electronic mass. It is an unstable particle. Its mean life is of the order of 10^{-8} second. Its symbol is π^+
2. Negative pi-mesons: it is a negatively charged particle whose charge is equal to the electronic charge and whose mass is 274 times the electronic mass. Its mean life is of the order of 10^{-8} second. Its symbol is π^-
3. Neutral pi-mesons: This particle has no charge. Its mass is nearly 264 times the electronic mass. Its mean life is of the order of 10^{-15} second. Its symbol is π^0. On disintegration, it forms two γ-photons.

$$\pi^0 \rightarrow \gamma + \overline{\gamma}$$

Photons:

There are the bundles of electromagnetic energy and travel with the speed of light. If the frequency of waves be γ, then the energy of a photon is hγ and momentum is hγ/c. Its symbol is γ.

All the elementary particles are grouped into three broad classes, baryons, mesons and leptons. The chief characteristics of the first two is that they are subject to strong nuclear interaction. Such particles are called hadrons. The leptons on the other hand are not subjected to strong nuclear interaction. They are subject to weak nuclear interaction. The particles belonging to all the groups are of course subject to gravitational interactions, if they have mass. Besides, the charged particles in all the groups are subject to electromagnetic interaction.

11.5 Quantum numbers (charge, spin, parity, iso-spin, strangeness etc)

The quarks have quantum numbers. The s-quark has a quantum number called strangeness. Like strangeness, the C,B and T quantum numbers are conserved in the strong and electromagnetic interactions and change by one unit in weak interactions. This means that the number of quarks minus antiquarks for each s, c, b and t must remain constant in strong and electromagnetic interactions, whereas in the weak interaction there is a change of quark flavor with the preferred sequences t→b→c→s, Since there of the quarks are needed to make a baryon, therefore, the baryon number is 1/3 for all the quarks. The isospin quantum number T is ½ and therefore, Ts=1/2 and -1/2 for the up and down quark respectively. The quantum number S of strange quark and B of beauty quark is -1. It is 1 for the charm and top quarks. The quark quantum numbers are summarized in the following table. These are related with charge Q and isospin component T, with the relation

$$Q = T3+(B+S+C+B+T)/2$$

Particles	Mass (MeV)	charge	strangeness
Proton	938.280	+e	0
Neutron	939.573	0	0
Lambda	1115.60	0	-1
sigma	1189.36	+e	-1
Xi	1314.9	0	-2
Omega	1672.5	-e	-3
pion	139.57	+e	0
Kaon	493.67	+e	+1
Eta	548.8	0	0
Electron	0.511	-e	-
Muon	105.66	-e	--

Tauon	1784.2	-e	-
Electron neutrino	< 35 eV	0	-
Muon neutrino	<0.52 MeV	0	-
Tauon neutrino	<150 MeV	0	-
photon	0	0	--
Graviton	0	0	-

11.6 LET US SUM UP:

- The gravitational force is an attractive one between every pair of nucleons, but it is smaller by a factor of 10^{39} than the electrical force between the two protons.
- The particle directory is discussed.
- The quarks have quantum numbers. The s-quark has a quantum number called strangeness. Like strangeness, the C, B and T quantum numbers are conserved in the strong and electromagnetic interactions and change by one unit in weak interactions.

11.7 Review questions:

1. Write a note on quantum number based particle classification.
2. What are the fundmental forces in nature? Explain.

11.8 Further readings:

1. Physics of the nucleus – Gupta and Roy, Arunabha Sen publishers, Kolkata.

2. Modern Atomic and Nuclear Physics- A.B.Gupta, Arunabha Sen pub. Kolkata.

UNIT XII CLASSIFICATION OF ELEMENTARY PARTICLES

Structure

12. 1 Introduction:

The discovery of various elementary particles was followed by an extensive study of their characteristic properties. This led to the determination of their mass, charge, life-time, quantum numbers, decay schemes, interactions etc.,

12.2 Objectives: Leptons, Baryons and quarks are discussed- Spin and parity assignments are deliberated- Iso-spin is defined- strangeness is discussed.

12.3 Leptons:

Leptons are weakly interacting particles. Charged leptons also show electromagnetic interaction. There are up to now twelve leptons: electron (e-), positron (e^+), pair of muons (μ^+, μ^-) pair of tauons (τ^+, τ^-) and three neutrions: electron-neutrino (ve), muon-neutrino (vμ) and tauon neutrino (ν_τ), and their antiparticles (antineutrinos : $\bar{\nu}_e, \bar{\nu}_\mu and \bar{\nu}_\tau$).

Leptons appear to have no internal structure and are point particles.

Electrons, the first of the elementary particles discovered, are negatively charged, stable, having a rest mass 9.11×10^{-31} kg and a rest energy 0.511 MeV/c^2. It is a spin ½ particles. Positron is an antiparticle of electron and hence is also called positive electron.

Muons were first observed in cosmic radiation and have a mass of about 200 me. Muons are also spin ½ particles.

Tauons have comparatively recently been discovered, having a large mass (1784.2 MeV/c^2) –heavier than nucleons. They are unstable and decay into muons and two neutrinos.

$$\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$$

Neutrinos were first discovered in β-emission, and therefore other neutrinos were obtained in other nuclear reactions.

All neutrinos are stable, having zero charge, spin ½ and negligible (or zero) mass.

All the Leptons interact weakly with other particles and they are all fermions as they obey Fermi-Dirac statistics.

12.4 Baryons:

Hadrons of half-integer spins are called baryons. Baryons have rest mass intermediate between that of a nucleon and a deuteron. These include the nucleons-protons and neutrons-and the hyperons that include lambda (λ^0), singma's ($\sum^{\pm}, 0$), Xi's ($\Xi^{-,0}$) and omega (Ω^-) particles, and their antiparticles.

Proton is stable (?), a free neutron is unstable with a mean life of 640s and decays into a proton, an electron and an electron-antineutrino.

Hyperons are strange particles, unstable and were first discovered in cosmic rays.

Baryons have even parity and they are fermions obeying the FD-statistics. It is believed that the total number of baryons in the universe is a conserved quantity.

12.5 Quarks:

Quarks are the basic constituent particles of which elementary particles are now believed to be composed. Many phenomena in particle physics have been predicted and explained theoretically with the concept of quarks. However, the experimental observation of free quarks remain ambiguous. There were at least three factors which made a quark model plausible in the 1960s. (i) The classic electron-proton elastic scattering experiments which demonstrated a proton with a finite form factor. This showed a finite radial extent of the electric charge and magnetic moment distributions. (ii) The hadron spectroscopy revealed in order and symmetry among the states of hadronic matter that could be interpreted with the symmetry laws. (iii) The deep inelastic scattering of electrons on protons revealed from factors corresponding to point like quasi-free constituents (called partons) of the hadrons such as the proton.

Until 1974, only three flavours of quarks were known, two or very nearly equal mass, called up quark (u) and the down quark (d). The proton, neutron and pi-mesons are composed of these two quarks. The third, more massive quark is called strange quark(s). It is a constituent of strange particles such as K-mesons, Λ^0 –hyperons etc, the u and d quarks form an isospin doublet, they are alike except for T_3 and Q, while the s-quark is an isospin singlet since no other quark is found to possess strangeness. Mesons are composed of a quark and antiquark pair, the antiparticles is formed by the antiquarks of those forming the particle.

In 1974, the discovery of unusual elementary particles (J/ψ meson) with lifetimes about 1000 times longer than typical mesons which decay through the strong interaction gave the direct evidence for the new quark. The new quark has to possess a property like strangeness which was called 'Charm' or c-quark. In 1977, at Fermi National Accelerator Lab near Chicago, L.M.Lederman and his colleagues reported a new resonance (upsilon particle at about 9.4 GeV and suggested it as the lowest lying state of a new quark system. This new quark was named 'b' for bottom (or beauty). Physicists expect a similar heavier quark which has the same relationship to the b-quark that the charm quark has to the strange quark. In 1984, a CERN group reported that the observed state with a mass of 30 to 50 GeV/c^2 might be a meson containing this heavier quark named t quark. The t may stand for top (or truth).

The up, charm and top (u, c and t) flavours each carry a positive electric charge equal in magnitude to two thirds that of the electron (2/3 e) and the down, strange and bottom (d ,s and b) flavors have a negative charge one third that of the electron (-1/3 e). These two groups are also called as up quarks (u, c and t) with charge (2/3 e) and the down quarks (d, s and b) with charge -1/3 e. The quarks have a spin angular momentum of one half unit of $\hbar$. They obey Fermi-Dirac statistics. As for interacting particles, no two quarks within a particular system can have exactly the same quantum numbers. Since the Ω-particle is made of three strange quarks, thus seems to violate the Pauli exclusion principle.

O.W.Greenberg in 1964 suggested that the each quark type (u, d and s) comes in three varieties identical in all measurable qualities but different in an additional property, which is known as color. The color hypothesis thus triplets the number of quarks. It triplets the rate of decay of π^{0-}meson and the total production cross-section for baryons and mesons in electron-positron annihilation. The experimental result at energies between 2 GeV and 3 GeV is in reasonable agreement with this color hypothesis.

12.5.1 Quark masses:

Among the six quarks, the least massive members are the u-and d-quarks, each of same mass, around 0.39 MeV/c^2, the lightest baryons, nucleons,Δ-particles, and the lightest mesons, pions must therefore be exclusively made of these two quarks. The s-quark is more massive, around 0.51 GeV/c^2. It carries a quantum number called strangeness and is therefore a necessary constituent of particles called strange particles (with non-zero strangeness), such as K-mesons, and baryon Λ. The c-quark is even more massive, having rest mass around 1.65 GeV/c^2. The b-quark has a rest mass about 5GeV/c^2. The t-quark is even heavier, having rest mass about 30 GeV/c^2. The t quark has been confirmed experimentally for its creation. It is also possible to discover a fourth generation of quarks, beyond these three groups.

12.6 Spin and parity assignments:

Spins and parities have been assigned to the low-lying energy levels in the very neutron-rich Z=45 nucleus ^{113}Rh identified in beta decay of the exotic fission fragment ^{113}Ru. The assignments are based on the experimental β intensities and log f_t values, gamma energies, intensities, branching and multi-polarities deduced from ^{113}Ru decay, taken together with the trends for the analogous levels and their comparative feedings and decays in the lighter n-rich Rh isotopes with A=105-111, and the supporting model considerations. No abrupt pattern changes are noticed for this N=68 nucleus lying just beyond the N=66 mid shell point. Whereas the low-lying positive parity level energies show very little variation or a slow decreasing trend, the negative parity level energies are seen to rise sharply with the addition of successive neutron pairs. The intruder band, having reached minimum energy in the N=64 isotope^{109}Rh, appears to have a parabolic increase on either side.

This expectation is borne out 25 by the fact that all the odd-A n-rich Rh isotopes with A=105-117 have ground state spin parity $I_\pi=7/2^+$ corresponding to I=j-1 for $(g^{9/2})^5$ valence protons, in accordance with the criterion of Bohr and Mottelson. Further, this survey reveals the fact that, while detailed spin-parity assignments and specific level structures have been reported for all the A ≤ 111 Rh isotopes 6-18, not even one of the 11 identified levels in the N=68 isotope ^{113}Rh has been given any spin-parity I_π assignment 6,7,19. Such information and the level classifications obtained thereby for this nucleus, lying just beyond the neutron (N=50 and 82) closed shells midpoint at N=66, is of a great interest.

12.7 Iso spin:

In any nuclear reaction, the total electric charge is conserved. Since elementary particles carry a charge ±1e and 0e. Heisenberg was the first to apply a quantum number to charge and referred to the concept as ‘iso spin’ it has nothing to do with spin or angular momentum.

It arose from the idea that pairs of particles like nucleons and triplets like pions hardly differ in their mass and may be considered as isotopes and that their charges, differing from each other by unity, suggest space quantization similar to electron spin and orbit in a magnetic field.

A multiplet number M is therefore assigned to such particles to indicate the number of their different charge states. For instance, for nucleons (protons and neutrons) the multiple number M=2. Similarly, for pions, M=3; for kaons, M=2 (and also for antikaons); M=1 for η_0; for sigma hyperons, M=3 etc.,

As in multiplicity of atomic energy states due to spin, the total number of states is (2s+1) where s is spin quantum number of electron, we may define iso spin quantum number I from the relation

$$M=2I+1 = I = (M-1)/2 \quad \text{-----------------(1)}$$

Isospin is treated as a vector $\bar{I}$ of magnitude $\sqrt{I(I+1)}$, like the angular momentum, but I is dimensionless. Its component in a particular direction, called z-axis, is given by I_3 (or I_2) which have the allowed values

$$I, (I-1), (I-2) \ \ldots\ldots\ldots\ldots\ldots\ldots.., -I \quad \text{-------------(2)}$$

For nucleus (M=2), I=(M-1)/2 =1/2, with the two components I_3=+1/2 assigned to proton and I_3=-1/2 assigned to neutron. Similarly, for pions (M=3), I=(M-1)/2=1. Hence I_3=+1, 0, -1; I_3=+1 is assigned to π^+, I_3=0 to π^0 and I_3=-1 to π^-

12.8 Strangeness:

Strangeness is another quantum number, first proposed by Pais and developed by Gell-Mann and Nishijima, to understand the strange behavior of K-mesons and hyperons (strange particles). Experimentally, it is found that K-mesons and Λ, $\sum$, Ξ and Ω hyperons are produced by strong interaction in high energy nucleon-nucleon collisions, but they decay, contrary to the expectation, only reluctantly by weak interaction having a life time $\geq 10^{-23}$ s. The inconsistency was resolved by the discovery of associated production-the strange particles being produced always in pairs.

Strangeness number S is defined as the difference of the hypercharge Y and the baryon number B.

$$S = Y-B \quad \text{-----------------------(1)}$$

$$Y= B+S \quad \text{----------------------(2)}$$

i.e., the hypercharge is the sum of the baryon number and the strangeness number

$$Q = I_3 + \frac{Y}{2} = I_3 + \frac{B+S}{2}$$

With this relation, the strangeness of particle will be an integer. Strangeness of an antiparticle is equal and opposite in sign to its associated particle.

For K^+ meson Q=+1, I_3=+1/2, B=0, S=+1

For Λ^0 and Σ^0 hyperons: Q=0, I_3=0, B=1, S=-1

For ϵ^- hyperon, Q=-1, I_3=-1/2, B=1 S=-2 etc.,

The value of S or various strange particles are given as under.

S=1, for particles K+, K0, $\overline{\Lambda}^0$, $\overline{\Sigma}^+$, $\overline{\Sigma}^0$, $\overline{\Sigma}^-$

S=2 , for particles $\overline{\Xi}^0$, Ξ^+

S=-2, for particles Ξ^0, Ξ^-

S= -1 for particles K-, $\overline{K}^0$, Λ^0, Σ^+, Σ^0, Σ^-

S=0 for other particles which are not strange.

12.9 LET US SUM UP:

- Leptons are weakly interacting particles. Charged leptons also show electromagnetic interaction.
- Hadrons of half-integer spins are called baryons. Baryons have rest mass intermediate between that of a nucleon and a deuteron.
- Quarks are the basic constituent particles of which elementary particles are now believed to be composed.
- Heisenberg was the first to apply a quantum number to charge and referred to the concept as ‘iso spin’ it has nothing to do with spin or angular momentum.
- Strangeness is another quantum number.

12.10 Review questions:

1. Write a note on Leptons
2. Discuss about Baryons.
3. Write a detailed note on Quarks.
4. What is isospin? Explain.
5. Define Strangeness – Explain

12.11 Further reading:

1. Nuclear Physics-V.Devananthan
2. Nuclear physics- D.C.Tayal

UNIT XIII GEL-MANN-NISHIJIMA RELATION

Structure:

13.1 Introduction:

The fundamental interactions, also known as fundamental forces, are the interactions that do not perform to be reducible to more basic interactions. There are four fundamental interactions known to exist: the gravitational and electromagnetic interactions, which yield noteworthy long-range forces whose effects can be seen directly in everyday life and the strong and weak interactions, which produce forces at minuscule, subatomic distances and manage nuclear interactions.

13.2 Objectives: The fundamental interactions is discussed- Translation in space is discussed- Rotations in space is discussed- SU(2) and SU (3) groups are deliberated- Charge conjugation is discussed- Parity is discussed- Gell-Mann-Nishijima formula is discussed.

13.3 The fundamental interaction:

The number of elementary particles so far discovered is very large, exceeding 200. Of these, only a few are stable, such as proton, electron, positron, neutrino and photon. The rest are all unstable of these, some have mean lives to decay much longer than the characteristic nuclear time. Most of these decay by weak interaction with mean lives extends from 10^{-13} second to several minutes (in the case of neutrons). A few like the $\pi 0$ meson and $\sum^0$ hyperon decay by electromagnetic interaction with mean lives$\sim 10^{-20}$ to 10^{-16} s. All these particles, suffering both weak and electromagnetic decays, are called semi-stable.

Apart from the stable and semi stable particles, a large number of particles has been discovered which decay by strong interaction with mean lives of the order of the characteristic nuclear time ($\sim 10^{-29}$ s). They are really unstable particles and are known as particle resonances.

To comprehend the underlying theoretical significance of the existence of such a large number of elementary particles, we shall first discuss about the fundamental interactions, which act between the elementary particles of which all matter is composed.

It is known that there are four different types of fundamental interactions in nature, which govern the behaviors of all observable physical systems. There are gravitational, electromagnetic, weak nuclear and strong nuclear interactions.

Gravitational force acts between all bodies having mass. It is described by the long range inverse square type. Newtonian laws of gravitation, later broadened by Einstein in his General theory of Relativity, which describes the gravitational interaction in terms of the curvature of space. The dimensionless quantity $Gm_e m_p/hc = 3\times 10^{-42}$ is usually taken as constant, characterizing this interaction, G being gravitational constant. The gravitational interaction between two bodies is believed to be mediated through the quantum of this interaction called the graviton, which has not been discovered as yet. So far as the elementary particles are concerned, the gravitational interaction between them can be entirely left out of consideration, because it is very weak compared to the other interaction.

Electromagnetic interaction is much stronger than the gravitational interaction. It is also a long range reverse square type interaction. Its strength is deformed by Summerfield's fine structure constant $\alpha = e^2/4\pi\varepsilon_0\hbar \sim 1/137$. The electromagnetic interaction between a proton and an electron is about 1037 times stronger than the gravitational force between them at the same distance. In modern quantum field theory, the electromagnetic interaction between charged particles is described in terms of exchange of virtual photons, which are the quanta of this field. The electromagnetic interaction manifested in the chemical behavior of the atoms and molecules. Rutherford scattering and so forth.

The third of fundamental interaction is the weak nuclear interaction which is responsible for the nuclear beta decay and the weak decay of certain elementary particles like the muons, pions, K mesons and some hyperons. Unlike gravitational or electromagnetic interaction, the weak interaction is a very short range the range being given by $\hbar/m_w C \sim 2.4 \times 10^{-18}$ m. Here mw is the mass of the w boson, mediating weak force. The coupling constant of the weak interaction has a value $g = 1.4 \times 10^{-42}$ Jm^3. When expressed in dimensionless from the weak interaction constant

g_w is found to be small compared to the fine structure constant α∼ 1/137. Here g_w= ($g/\hbar c$) (ƛ)2 ƛ =λ/2π, λc being Compton wavelength for the electron. Comparision of the characteristic times for the operation of the weak (10^{-10} S) and strong (∼ 10^{-23}S) interactions give the ratio of the former to the latter ∼ 10^{-13}. The weak interaction is mediated through the heavy vector becomes, known as W^x and Z^0 bosons discovered by G.Amison et al using the p $\bar{p}$ colliding beam at the C.M energy of 540 GeV obtained from SPS facility at Cern in1983. They have the masses,

$m_w c^2$=80.9 GeV

$m_z c^2$=93 GeV

An important characteristic of the weak interaction, which distinguishes it from the other fundamental interactions, is that parity is not conserved in weak interaction.

The fourth fundamental interaction is the strong nuclear interaction between protons and neutrons, having short range like the weak nuclear interaction, the range being $\hbar/m_\pi$C= 10^{-15} m within this range it predominates over all the other forces between the neutron and the proton with a characteristic strength parameter α ∼0.3. This is much larger than the strength parameter α∼1/137 of the electromagnetic interaction. This time for the operation of the strong interaction is of the order of the characteristic nuclear time (∼10^{-23} S).

The strong interaction force is mediated through the exchange of a pi-meson ($\pi^{\pm}$, π^0), which is the quantum of the interaction field.

The properties of the strong nuclear force are manifested in the building of the complex nuclei. The quantitative connection between the nuclei forces and nuclear structures is not yet fully known.

An important method for the investigation of the nature of the strong nuclear force is the study of the properties of the hyper nuclei in which a nucleons in a nucleus is replaced by a hyperon. The first hyper nucleus was discovered in 1953. It was a boron nucleus in which a neutron was replaced by a Λ^0-hyperon. Later many other type nuclei were produced and their properties investigated. One such is Λ^0-nucleus of tritium, which decays according to the scheme Λ^3 H→ 3He+π^- for which q=41.5 MeV.

13.4 Translation in space:

Some classical invariance principles are related to the nature of space-time. Invariance of the Hamiltonian (the operator or expression for total energy) under a translation for an isolated, multi particle system leads directly to the conservation of the total momentum of the system. This can be demonstrated classically, but we will take a quantum mechanical approach, defining an operator $\hat{D}$ which produces a translation of the wave function through δx:

$$\hat{D}\psi(x) = \psi'(x) = \psi(x + \delta x).$$

Then it can be shown that $\hat{D}=\exp(i\hat{P}\delta x/\hbar)$ where $\hat{P}$ is the momentum operator.

$\hat{P}$ is said to act as a "generator of translations". Now since the energy of an isolated system cannot be affected by a translation of the whole system, $\hat{D}$ must commute with the Hamiltonian operator $\hat{H}$, i.e. $[\hat{D}, \hat{H}] = 0$; it must therefore also be true that $[\hat{P}, \hat{H}] = 0$, and so $\hat{P}$ has eigenvalues which are constants of the motion.

We therefore have three equivalent statements:

1. Momentum is conserved in an isolated system.
2. The Hamiltonian is invariant under spatial translations. (Equivalently, it is impossible to determine absolute positions.)
3. The momentum operator commutes with the Hamiltonian.

13.5 Rotations in space:

"In 1920s, Otto Stern and Walther Gerlach of the University of Hamburg in Germany showed a series of important atomic beam experiments. Knowing that all moving charges produce magnetic fields, they suggested to measure the magnetic fields produced by the electrons orbiting nuclei in atoms. Much to their surprise, however, the two physicists found that electrons themselves act as if they are spinning very rapidly, producing tiny magnetic fields independent of those from their orbital motions. Soon the terminology 'spin' was used to describe this specious rotation of subatomic particles.

"Spin is a bizarre physical quantity. It is analogous to the spin of a planet in that it gives a particle angular momentum and a tiny magnetic field called a magnetic moment. Based on the known sizes of subatomic particles, however, the surfaces of charged particles would have to be moving faster than the speed of light in order to produce the measured magnetic moments. Furthermore, spin is quantized, meaning that only certain discrete spins are allowed. This situation creates all sorts of complications that make spin one of the more challenging aspects of quantum mechanics.

"In a broader sense, spin is an essential property influencing the ordering of electrons and nuclei in atoms and molecules, giving it great physical significance in chemistry and solid-state physics. Spin is likewise an essential consideration in all interactions among subatomic particles, whether in high-energy particle beams, low-temperature fluids or the tenuous flow of particles from the sun known as the solar wind. Indeed, many if not most physical processes, ranging from the smallest nuclear scales to the largest astrophysical distances, depend greatly on interactions of subatomic particles and the spins of those particles." Spin is the total angular momentum, or *intrinsic* angular momentum, of a body. The spins of elementary particles are analogous to the spins of macroscopic bodies. In fact, the spin of a planet is the sum of the spins and the orbital angular momenta of all its elementary particles. So are the spins of other composite objects such as atoms, atomic nuclei and protons (which are made of quarks).

13.6 SU(2) symmetry:

A proton and a neutron are identical as far as the nuclear force is concerned but they are different in their electromagnetic interactions. A set of symmetry operations can be assumed which can transform proton into neutron or vice versa in the absence of an electromagnetic field. The existence of such symmetry implies that something remains constant under strong interactions. This is known as isospin and is ½ for proton -1/2 for neutron. The operator of the symmetry group change the co-ordinates of the isospin in such a way that it reverses the sign of T_3. The particular symmetry group applicable to isospin conservation is a form of unitary symmetry group applicable to isospin conservation is a form of unitary symmetry known as U(2) which can be expressed by a set of (2×2) matrices. This group may be reduced to a special unitary group SU(2); which has three operators. The two dimensions refer to the basic state.

By the use of the absence of SU(2) group it can be shown that all inducible representations of the symmetry group consists of a multiplet of (2T+1) states. All the members of the multiplet would differ in charge and T3, SU(2) symmetry is violated by the electromagnetic interaction for which conservation of isospin is not applicable.

If P and n are the proton and neutron states then

$$\binom{p}{n}.(\bar{p}\ \bar{n}) = \frac{p\bar{p}+n\bar{n}}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}(p\bar{p}-n\bar{n})\,{}^{1}/_{2} & p\bar{n}\\ n\bar{p} & (p\bar{p}-n\bar{n})\,{}^{1}/_{2}\end{pmatrix}$$

The first term of RHS represents singlet η-meson, T=0, J=$\bar{0}$). The second can be written in an array form with normalizing factors

$$\begin{pmatrix}\frac{\pi^0}{\sqrt{2}} & \pi^+\\ \pi^- & \frac{\pi^0}{\sqrt{2}}\end{pmatrix}$$

Under the SU(2) group operations, the particle state transform into each other within these multiplets.

13.7 SU(3) symmetry:

M.Gell-Mann and independent by Y.Ne'eman in 1964, through a symmetry scheme known as the eight fole way in which all the known particles and resonances are grouped to form families containing 1 or 8 or10 or 27 members by the application of special unitary symmetry group known as SU(3). All the members of a family had the same spin and parity (same JP).

The group SU(3) is the group of all unimodular, unitary, 3×3 matrices, i.e., the group of all 3×3 matrices U, such that U is unitary, i.e $U^{\oplus}$ =U^{-1} and unimodular, i.e., determined U=1 where $U^{\oplus}$ is the Hermitian adjoint of U and is obtained by interchanging rows and columns and taking the complex conjugate of each element, i.e.,

$$(U^{\oplus}) = U_{ij}^{\oplus}$$

SU stands for special unitary. The term special refers to the unimodularity condition.

The SU (3) theory is a generalization of the isospin theory which in group theoretical language is classified as SU(2) theory which deals with eight dimensions.

It is called the eight-fold way because it covers relations between eight conserved quantities as also it recalls a saying by lord Buddha, which reads as under.

Now this, O monks, is the noble truth that leads to the cessation of pairs this is Lord Buddha's noble Eight-fold way to nirwana namely.

(1) right views (ii) right intention (iii) right speech (iv) right action (v) right effort (vi) right living (vii) right mindfulness (viii) right concentration

The 8 and 10 member groups of this theory are of particular interest. These are baryon octet, the meson octet and baryon decuplet. The baryon octet group of the theory is as follows.

We have seen that the strongly interacting particles of hadrons, characterized by the values of their masses, spin partiy Jp, Baryon number B, isospin I, third component of iso-spin I_3 and hypercharge Y, can be grouped quite naturally into charge multiplets called iso-multiplets within which the values of I_3 vary from +I through 0 to –I, but all the other characteristics are the same except for small mass differences. For example in case of the nucleon isospin doublet, we have,

Jp=1/2+, B=1, I=1/2, Y=1 and we have I_3= +1/2 for the proton and I3=-1/2 for the neutron.

The nuclear masses are

M_p=938.2 MeV and m_n=939.6 MeV

So mean nucleon-mass m_N= ½ (m_p+m_n)=939 MeV and the mass difference of the nucleon is:

M_n-m_p=1.4 MeV

Therefore, it is seen that the mass difference of the nucleons is quite small as compared to mean nucleonic mass. It then appears possible to arrange hadrons (strongly interacting particles) into larger multiplets than the iso-spin multiplets, in which spin parity Jp and the Baryon number B are the same but the hypercharge Y, isospin I and I3 vary for the various numbers of the multiplet. For example, let us consider the low mass Baryons with Jp=1/2 +. There are eight in number (p, n, Λ^0, Σ^+, Σ^-, $\in^0$, $\in^-$). The values of the various quantum numbers for them are given in the following table

In case of ⋀0- that next Jp= ½+ Baryon, we have B=1, Y=0, I=0 and I_3=0. Mass of Λ^0 is given by:

m_Λ^0 =1115.6 MeV= 1116 MeV

Hence, m_Λ^0- mN= (1116-939)=177 MeV

In case of ∑-triplet we have B=1, Y=0, I=1 and I_s=-1 for Σ^-, I3=0 for Σ^0 and I_3=+1 for Σ^+ and their masses are

m_Σ^- =1189.5 MeV

m_Σ^0 =1192.5 MeV

m_Σ^- =1197.3 MeV

So their mean mass m_Σ=1193 MeV

And $$m_\Sigma^- - m_\Sigma^0 = 4.8\ MeV$$

$$m_\Sigma^0 - m_\Sigma^- = 3.0\ MeV$$

$$m_\Sigma^0 - m_\Lambda^0 = (1193 - 1116) = 77\ MeV$$

In order of increasing mass, the next Jp=1/2 + baryons are Ξ^- and Ξ^0 hyperons. They have B=1, Y=-1, I=1/2 and I3=-1/2 for Ξ^- and I_3=1/2 for Ξ^0

Their masses are:

$m_\in^0$ =1315 MeV, $m_\in^-$= 1321 MeV

So their mean mass $m_\in^0$=1318 MeV

$m_{\in}^{-}$- $m_{\in}^{0}$ = (1321-1315)=6 MeV

$m_{\in}^{0} - m_{\Sigma}$ =(1318-1193)=125 MeV

The eight-fold way arrangement of this Baryon octet is shown in fig.13.1

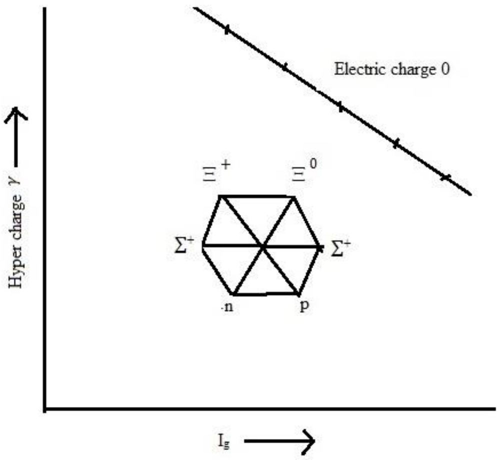

Fig.13.1

The existence of this multiplet and other hadron multiplets which are made up of several isospin multiplets, leads to the thinking that there is some ‘super strong’ interaction while by itself will yield the same mass for all the members of the multiplet. The difference in masses of the members of a SU(3) mulitplet is considered to be due to some symmetry breaking perturbation which depends upon Y and I.

13.8 Charge conjugation:

Charge conjugation is the operation which changes the sign of the charge of a particle without affecting any of the properties unrelated to charge. It may be defined as the transformation between a particle and its antiparticle. It has been found experimentally that the operation of both the strong and electromagnetic interactions is invariant to charge conjugation. For instance, such invariance is found experimentally in the strong interaction annihilation of a proton and an antiproton into the particle antiparticle pair K^+K^- and is also found in measurements of the electromagnetic decay of the η_0-meson. We thus believe that the nucleus of the anti-deuterium atom (strong interaction behavior) and also the positron (electromagnetic interaction behavior) would act in the same way. The charge conjugation is not conserved in the weak interaction, i.e., the weak interaction does distinguish between a system and its charge conjugate. The charge conjugate does not simply mean a change over the opposite electric charge or magnetic moment, the spin of other charge quantum numbers [hypercharge Y, baryon number B, Lepton number (Ie, Ip) is also reversed without changing mass M and spin s. Thus a unitary operator, also known as charge conjugation operator C, satisfies the following relations:

CQC^{-1}=-Q, CYC^{-1}=-Y, CBC^{-1}=-I_e and CI_pC^{-1}= -I_p ---------------------(1)

Some elementary particles e.g., γ, π^0, η^0 –mesons and the positronium atom (e^-+e^+) are transformed into themselves by charge conjugation. They are their own anti-particles. These are known as self-conjugate or true neutral particles. The neutron (B=1, Y=1) and K^0-mesons (Y=1, B=0) are not invariant under C.

A system is said to possess charge conjugation symmetry or to be invariant under charge conjugation if the system (or the process) is such that it is impossible to know that it has undergone charge conjugation. For example, the operation C converts the negative pion decay ($\pi^- \rightarrow \mu^- + \bar{\gamma}_\mu$) into the positive pion decay ($\pi^+ \rightarrow \mu^+ + \gamma_\mu$), since the π+ is the anti-particle of π-. Until about four decades ago it was believed that the entire universe is invariant under C. In turned out that the μ+ and μ- decay electrons have angular distributions of opposite symmetry: that the π^+ and π^- decay moves have opposite polarizations and that while a free neutrino is left handed an anti-neutrino is right handed. Charge conjugation applied to a free moving neutrino then results in a process which does not exist in nature.

Positronium is a hydrogen like bound state of an electron and a positron. According to the Dirac's theory the charge conjugate, which corresponds to the wave function of positrons, has the form

$$C= ie^{i\theta}\begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix} = e^{i\theta}\, i\alpha_y \quad \text{---------------------------------(2)}$$

The phase φ is arbitrary. For zero phase (φ=0)

$$C=i\alpha_y$$

Where α_y is a square and σ_y is the pauli spin matrix having value

$$\sigma_y=\begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \quad \text{--(3)}$$

For single photon state

$$C|n\gamma> \quad \text{--(4)}$$

For a system of n-photons

$$C|n\gamma> = (-1)n\ |n\gamma>.$$

Thus the n-photons state is also in an eigenstate of C with the eigen value (-1)n. Since π0 mesons decay through electromagnetic interaction into two photons $\pi^0 \rightarrow 2\gamma$, it follows that the π0-meson is self conjugate and that $|\pi^0>$ is an eigenstate of C with eigen value +1, i.e.,

$C\ |\pi^0> = |\pi^0>$

On the other hand $|\pi^+>$ and $|\pi^->$are not eignestates of C as

$C|\pi^+> = -|\pi^->$ and $C\ |\pi^-> = -\ |\pi^+>$

As the triplet state is symmetrical and the singlet spin states anti-symmetrical, hence to exchange an electron with a positron we must introduce a factor $(-1)^{x+1}$ as well as a factor (-1)l. Thus we have

$(-1^{)\ l+s+1}$ C= -1 or $C=(-1)^{l+s}$ --------------------------------(5)

And the positronium atom is in an eigen state of C with eigen value $C=(-1)^{\ l+s}$

Above relation gives that the singlet ground state (l=1) decays into two photons and the triplet ground state decays into three photons. Similarly a system composed of two identical bosons of opposite charges, such as π+, $\pi^{-,}$is in an eigenstate of C. Since the wave function of two bosons remains unaltered under the exchange of the two particles including charge, the eigen value of C is given by C=1.

13.9 Parity:

The parity principle says that there is symmetry between the world and its mirror image. This may be defined as reflection of every point in space through the origin of a co-ordinate system $x\rightarrow -x$, $y\rightarrow -y$, $z\rightarrow -z$ or $r \rightarrow -r$. If a system or process is such that its mirror image is impossible to obtain in nature, the system or process is said to violate the law of parity conservation.

Human body is a good example of mirror symmetry. The body of a cat is symmetric except for the position of the steering wheel. If we were looking at the mirror image of a normal car, it seems to violate the symmetry but it is not the case as it is also possible to design a car with the steering wheel on the other side. The mirror view of a printer could design inverted type and produced a page. One can read the page from right to left. The type of printing is not unnatural but is unconventional and unfamiliar. The value of the physical quantities of classical physics either remain unaltered or change sign under space inversion. Thus the quantities may have either even or odd intrinsic parity. The ordinary scalars such as temperature, electric charge, energy have been parity. The ordinary vectors such as momentum, force, electric field have odd parity. Pseudovectors such as angular momentum, magnetic filed, behave as vectors under rotations but have the even parity. Similarly pseudo scalars (the product of any ordinary polar vector with a pseudovector) have the odd parity).

All phenomena involving strong and electromagnetic interactions alone do conserve parity. In these cases the systems can be classified by the eigen values of the parity operator P. For a single particle Schrodinger wave function ψ, the result of the parity operation is

$$P|\psi(x)> = e^{i\alpha}\,|\psi(-x)> \qquad \text{--------------------------(6)}$$

As α is an arbitrary real phase, hence can be set equal to zero

$$P|\psi(x)> = |\psi(-x)> \qquad \text{--------------------------(7)}$$

And $$P^2\,|\psi(x)> = |\psi(x)> \qquad \text{-------------------------(8)}$$

It shows that eigenvalues of P may be +1 or -1.

The parity of the photon depends upon the mode of transition, it is due to the change of the sign of electromagnetic current j under the parity operation. The nucleons and electrons are assigned positive (or ever) intrinsic parity. The pions have negative (or odd) as they involve in strong interactions with nucleons. K-mesons and $\eta 0$ –meson have anegative parity. $\wedge^0$, ϵ-, $\sum$, Ω –hyperons have positive intrinsic parity. All anti-particles of spin ½ (the ferminons) are of opposite partity to the corresponding particle, while the bosons and their anti-particles have the same parity.

The conservation of parity requires that the Hamiltonain of a free system commute with the parity operator.

$$(PH-HP)=0 \qquad \text{-------------------------------------(9)}$$

The transition probability must be scalar, it may contain pseudo scalar operator (I.p). The conservation of parity prevents the mixing of even or odd operators in the amplitude. Thus for the non-conservation of parity in β-decay, the transition probability must contain both scalar and pseudoscalar terms or the number of electrons emitted parallel and anti-parallel to the spin of the source should be different.

The weak decay of the K-mesons, which was difficult to reconcile with parity conservation and known as the $\tau - \theta$ puzzle, was explained by Lee and Yang. They suggested that the weak interaction was not invariant to space reflection. In 1956, we and others, using polarized ^{60}Co nuclei, found that the direction of emission of electrons in the transformation to ^{60}Ni was preferentially opposite to the spin direction. The value of pseudo scalar I.p., where I is the nuclear spin and p the electron momentum, was measured and found to be different from 0.

13.10 GEL-MANN-NISHIJIMA formula:

Strangeness is another quantum number, first proposed by Pais and developed by Gell-Mann and Nishijima, to understand the strange behavior of K-mesons and hyperons (strange particles). Experimentally, it is found that K-mesons and Λ, $\sum$, Ξ and Ω hyperons are produced by strong interaction in high energy nucleon-nucleon collisions, but they decay, contrary to the expectation, only reluctantly by weak interaction having a life time$\geq 10^{-23}$ s. The inconsistency was resolved by the discovery of associated production-the strange particles being produced always in pairs.

Strangeness number S is defined as the difference of the hypercharge Y and the baryon number B.

$$S= Y-B \quad \text{-----------------------(1)}$$

$$Y=B+S \quad \text{----------------------(2)}$$

i.e., the hypercharge is the sum of the baryon number and the strangeness number\

$$Q= I_3+ \frac{Y}{2} = I_3+ \frac{B+S}{2}$$

With this relation, the strangeness of particle will be an integer. Strangeness of an antiparticle is equal and opposite in sign to its associated particle.

For K^+ meson Q=+1, I_3=+1/2, B=0, S=+1

For Λ^0 and $\sum^0$ hyperons: Q=0, I_3=0, B=1, S= -1

For Ξ^- hyperon, Q=-1, I_3=-1/2, B=1 S= -2 etc.,

The value of S or various strange particles are given as under.

S=1, for particles K+, K0, $\overline{\Lambda}^0$, $\overline{\sum}^+$, $\overline{\sum}^0$, $\overline{\sum}^-$

S=2 , for particles $\overline{\Xi}^0$, Ξ^+

S=-2, for particles Ξ^-

S= -1 for particles K-, $\overline{K}^0$, Λ^0, $\sum^+$, $\sum^0$, $\sum^-$

S=0 for other particles which are not strange.

13.11 Summary

- ❖ The strong interaction force is mediated through the exchange of a pi-meson ($\pi^{\pm}$, π^0), which is the quantum of the interaction field.
- ❖ Some classical invariance principles are related to the nature of space-time.

- ❖ Charge conjugation is the operation which changes the sign of the charge of a particle without affecting any of the properties unrelated to charge.
- ❖ The parity principle says that there is symmetry between the world and its mirror image.

13.12 Review questions:

1. What are the fundamental interaction?
2. Write a brief accout on Translation in space.
3. Explain SU(2) and SU(3) symmetry groups.
4. Obtain the Gel-Mann-Nishijima relation.
5. Write a detailed note on parity.
6. Discuss about the charge conjugation.

13.13 Further reading

1. Nuclear Physics- Irvin Kaplan, Oxford & I.B.H Pub&Co.
2. Introduction to Nuclear Physics- Herald Enge, Addision Wesley Pub.Co., U.S.A

UNIT XIV SYMMETRIES

Structure:

14.1 Introduction:

Identical particles, also called indistinguishable or imperceptible particles, are particles that cannot be renowned from one another, even in principle. Species of identical particles include, but are not limited to elementary particles such as electrons, composite subatomic particles such as atomic nuclei, as well as atoms and molecules.

14.2 Objective: Time reversal is discussed- CPT invariance is deliberated- Application of symmetry arguments in particle reaction is discussed- Parity non-conservation in weak interaction is discussed- Relativistic kinematics is discussed.

14.3 Time reversal:

Time reversal operator is defined as that operator which reverses the direction of time, or the direction of all motions. Under the operation displacement, acceleration and electric fields remain invariant but momenta, angular momenta and magnetic fields invert their signs. If the time reversed process is impossible to occur in nature we can say that the process violates time reversal symmetry. Time reversal changes the direction of flow of time, like running the movie of a phenomenon backwards. The result is usually strange.

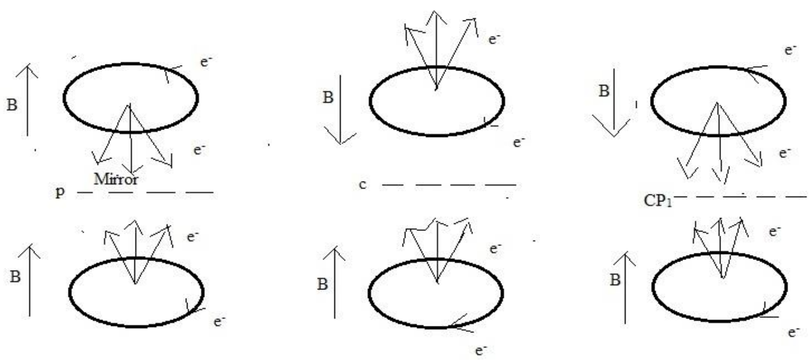

Fig. 14.1

Time reversal invariance field sits simplest application in the world of particles, where it appears to govern the strong and electromagnetic interactions and possibly also the weak. It also shows that a particle possessing time reversal symmetry cannot have electric and magnetic dipole moments simultaneously. The time reversal process is the creation of an electron-positron pair by the collision of two photons.

If time reversal operation is applied to decaying $\pi^+ \rightarrow \mu^+ + v_{\bar{\mu}}$. Under time reversal the momentum vectors are reversed and spins go around in the opposite directions. We have v_μ and μ^+ with the proper handedness interacting to form a pion.

Time reversal invariance is satisfied in quantum mechanics if the Hamiltonian H is time independent and real. In this case ψ^*(x,-t) is the time reversal wave function of ψ (x,t). Thus time reversal operation T changes ψ as,

$$T\,\psi\,(x,t) = \psi^*(x,-t)$$

Where the asterisk indicates the complex conjugate

The motion of a particle in an external fixed magnetic field is not invariant under inversion of time. The relativistic treatment of time reversal shows that the inversion of time axis inverts the sign of the electrostatic potential. The π0-mesonic field, like the magneto static potential, is odd under time reversal in order to ensure that the interaction is time reversible.

14.4 CPT invariance:

All interactions in nature are invariant under the joint operations of charge conjugation(C), parity operation (P) and the reversal of time (T). The orders of the three operations are immaterial.

The invariance of CPT transformation implies that if an interaction is not invariant under any one of the C, P or T-operations, Its effect must get compensated by the joint effect of the other two.

All evidences so far point to the universal validity of the theorem. But the discovery of a small but definite violation of CP-invariance by Chistensen et.al has opened up the operation of the universal validity of CPT-theorem, for if time-reversal invariance holds, then CPT theorem dictates that CP-invariance must also hold. The CPT violation, if any, can probably observed in the properties of K^0 and $\bar{K}^0$ mesons.

Combined inversion:

Landu (1956) advanced a hypothesis to the effect that any physical interaction must be invariant under simultaneous reversal of position coordinates and change over from particles to anti-particles. For example, a neutrino has a definite helicity and its parity conjugate has opposite helicity. The charge conjugate of the neutrino also has opposite helicity. Thus under the combined operation PC (or CP) the neutrino also has opposite helicity. Thus under the combined operation also known as combined parity (charge and space) is conserved in most of the known physical processes. Let us consider the decay of the positive pion

$$\pi+ \rightarrow \mu_L^+ + v_{\mu l}$$

Here subscript L indicates that neutrino and positive muon fly apart with left handed spin. As the C-inversion changes particles into anti-particles and vice-versa. Whereas the P-inversion converts left handed motion to right handed motion. Hence

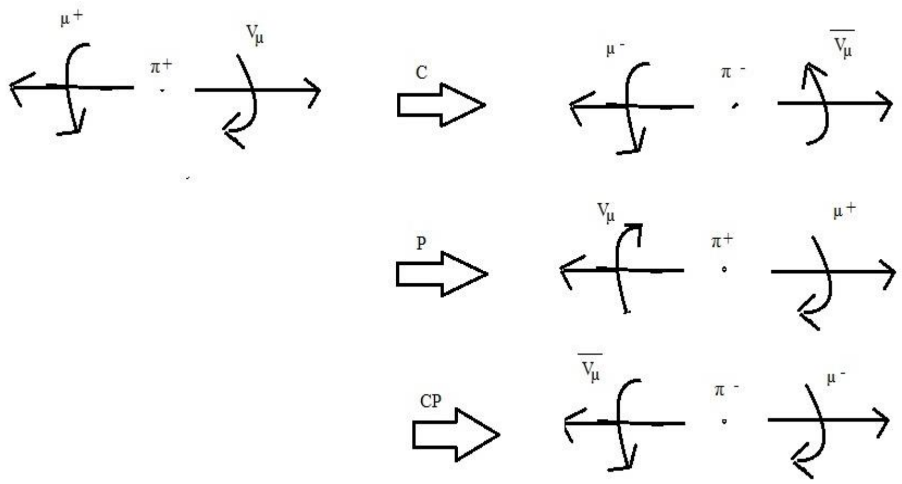

Fig.14.2

Since the fermions all have intrinsic parity (+1), while the corresponding antiparticles all have intrinsic parity (-1). For bosons the corresponding antiparticles has the same parity, thus π^+ and π^- mesons have the same intrinsic parity (-1). Thus the charge conjugation operator anticommutes with the parity operator when applied to fermions, but commutes with the parity operator when applied to bosons.

Let us consider the case of β-decay of polarized nuclei (e.g., ^{60}Co). The interpretation of the parity non-conservation, charge non-conservation and conservation under the combined operation is shown in Fig.14.2. In this figure, B shows the direction of a magnetic field due to current loop, used for polarizing the nuclei. It represents the nuclear spin and thus known as polarization vector. The upper diagrams represent the result of the reflection of the process shown in the lower diagram of Fig.14.2. Fig.14.2(a) shows that the space reflection creates a different system, as it changes the direction of decay arrows but not the creates a different system, as it changes the direction of decay arrows but not the direction of B. Fig14.2(b) represents the charge conjugates only. This process leaves the direction of the decay unchanged, although electrons are replaced by positrons and the polarization direction is reversed. Fig

2(c) shows the combined effect of CP operation. This shows that under CP reflection, we obtain the process of decay of the antinucleus. From the above results we conclude that the type of symmetry can be obtained by the combined operation of C and P only in weak interactions. Number of other examples show that the weak interactions do not grossly violate the combined CP-invariance. Unfortunately the decay of K-meson is not invariant under the combined operation of CP.

14.5 Parity non-conservation in weak interaction

A particle is moving with a large velocity can be quantum mechanically associated with a wave and the wave motion can be described by a wave function ψ (x,y,z) which depends on the space coordinates (x,y,z). Also ψ^* be the complex conjugate of ψ, $\psi^*\psi=|\psi|^2$ gives the probability of finding the particle at any given point. Parity is the property of such a wave function representing a quantum mechanical nuclear state, which may or may not change its sign on inversion of the space coordinates throughout from (x,y,z) to (-x,-y,-z) i.e., on reflection of the coordinate system at the origin. The parity of the nucleus is thus related to the behavior of nuclear wavefront as a result of reflection.

It was believed, till 1956 that is in all nuclear reactions, the parity was conserved. But in 1956, direct experimental evidence was obtained that parity is not conserved in nuclear phenomena involving weak interaction forces. The non-conservation of parity was first suspected theoretically by Li and Yang and was subsequently confirmed experimentally by Wu in 1956.

The weak interaction in β-decay provides an example on non-conservation of parity. The conservation of parity leads to some important selection rules in nuclear, atomic and molecular processes and in the production and decay of elementary particles. Parity is thus an important characteristic of a state describing quantum mechanical systems.

An interesting consequence of the fact that parity P is a good quantum number is that nuclei can have no permanent electric dipole moment.

While we are familiar with the classical definition of electric dipole moment of a charge distribution, quantum mechanically it is defined as

$$\langle D\rangle=\int \sum_j e_j \bar{r}_j \ldots\ldots|\bar{r}_1, \bar{r}_2, \ldots\ldots\ldots\ldots \bar{r}_n|^2 \, d\bar{r}_1 d\bar{r}_2 \ldots\ldots\ldots\ldots d\bar{r}_n \text{---(1)}$$

If now we replace $\bar{r}_1 = -\bar{r}_1$ $\bar{r}_2 = -\bar{r}_2$ etc., the first factor $\bar{r}_j$ changes sign, but not $|\psi|^2$ because of parity.

$<D> = - <D> = <D> = 0$

We thus obtain the important result that **the electric dipole moment of a nucleus in its ground state vanishes.** This is also true for all non-degenerate excited states of the nucleus.

14.6 Application of symmetry arguments to particle reaction:

We shall here only a brief reference to this symmetry classification. In 1962, M.Gell-Mann and Y.Ne'eman proposed independently an extension of the scheme of classification of elementary particles. The extension is based on the values of I_3 and the hyper charge Y=B+S) of the particles and is known as SU3-symmetry or the special Unitary Group of rank 3 symmetry. This is also called the octet symmetry or eight-fold way.

They proposed, in place of simple isotopic invariance as is assumed in SU2, the existence of a group of eight baryons, namely p,n, Λ, Σ^+, Σ^0, Σ^-, Ξ^- and Ξ^0 in a super-multiplet (octet) in SU3- scheme. All these baryons have $J^p = 1/2^+$. The different baryon groups differ in their values of either the isospin I or the strangeness number S or both. Their masses are also different but the mass-difference does not exceed 15 %.

It was therefore assumed that the above eight baryons, formed as a result of a common but very strong interaction, have essentially equal masses (eight-fold degeneracy in strangeness and charge). The octet symmetry however is broken by a moderately strong and strangeness-dependent interaction. And this would remove the strangeness S –values. Again, in each group of a given S-value, the change degeneracy is removed by the action of an electromagnetic interaction so that the components of same I but with different I_3-values have slightly different masses. The first splitting is of the order of $\Delta M/M \sim$ 10% to 20 %, while the second splitting is of the order of $\Delta M/M \sim$ 1%.

This scheme classification, based on octet symmetry, is better visualized if a graphical plot of the baryon octet in the I_3-Y plane is made. This is demonstrated in Fig. 14.3 and is called a weight diagram.

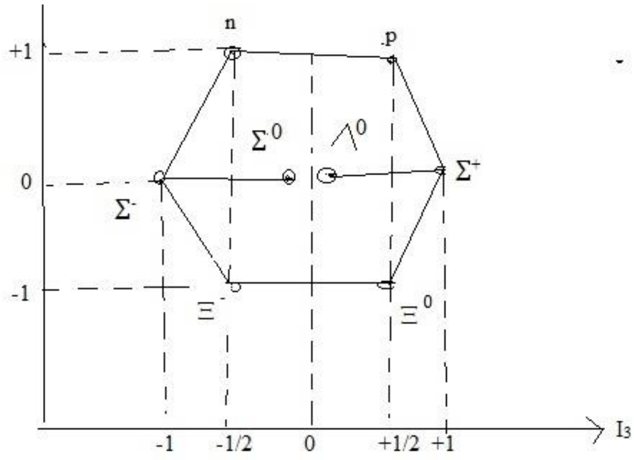

Fig.14.3

The members of the octet family form a symmetric hexagon (Fig.14.3) with one member (one baryon) at each corner and two at the centre. The plot shows that the two nucleons-p and n-with S=0 and $I_3=\pm\frac{1}{2}$ fall on the line Y=1. The three ∑-hyperons with S=-1 and $I_3=\pm 1,0$ fall in the line Y= -1. Finally, the single Λ -hyperon, with S=-1 and $I_3=0$ is at the centre of the hexagon, along with $\sum^0$-hyperon, with Y=0.

A similar super-multiple for mesons can also be arranged in a similar hexagon(fig.). The members of this octet family consists of two kaons (S=1) with $I_3=\pm$ ½, the three pions (S=0) with $I_3=\pm 1,0$, the two antikaons (S=-1) with $I_3 = \pm$ ½ and the isospin single η^0-meson ($I_3=0$, Y=0). While the two kaons will be on the line Y=1, the three pions on the line Y=0, the two antikaons on the line Y=-1, the η^0 ($I_3=0$) is at the centre of the hexagon with Y=0, along with $\pi 0$ (I3=0). The resonance particle η (958) is also included which makes the octet essentially a nonet.

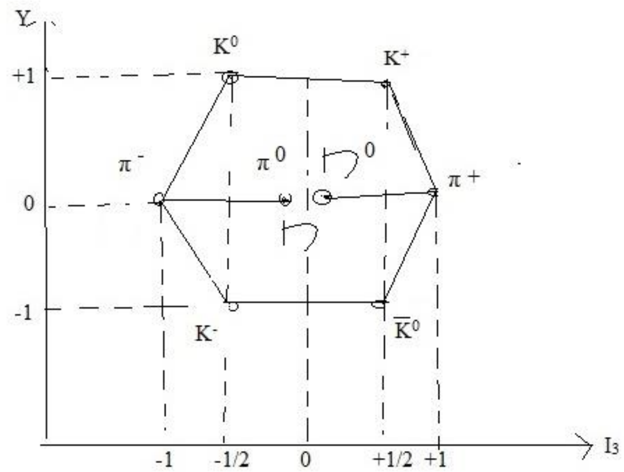

Fig.14.4

Similar weight diagrams in Y-I_3 plane can be constructed for resonance particles.

Apart from octet, the existence of a baryonic (resonance) decupled was also predicted by SU3-symmetry. Its weight diagram, as shown in Fig.14.5 is an equilateral triangle. The decuplet comprises an isotopic quadruplet, a triplet, a doublet and a singlet. The quadrupole Δ have S=0, Y=1 and I_3=3/2,1/2, -1/2, -3/2; the triplet ∑ have S=-1, Y=0 and I_3=1, 0, -1; the double Ξ with S=-3, Y=-2 and I_3=0

It is interesting to note when the prediction of the decuplet was made, the particle Ω- was not discovered. The predictions came true only subsequently which was considered as a triumph or the SU3 –symmetry. Even the properties of Ω-, as predicted by SU3-scheme, agreed and have been verified. The discovery of anti-omega (Ω-) particle has also been made.

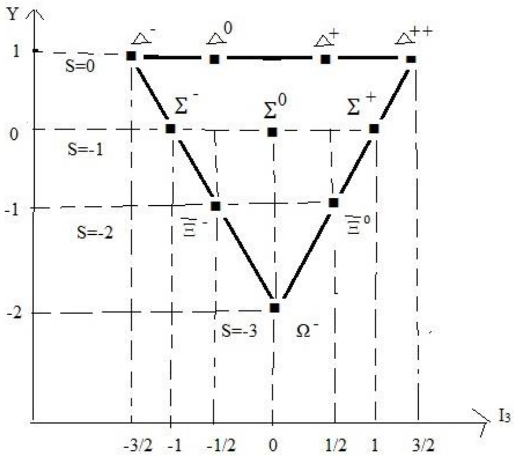

Fig.14.5

We shall however not go into the discovery of Ω- hyperons and the various predictions of SU3 –symmetry. While some of the predictions came true, some were not satisfied, hinting at the fact that unitary symmetry is much broader in scope than isotopic invariance, based on SU3-symmetry

14.7 Relativistic kinematics:

Many elementary particles are the product of high energy (relativistic) collisions. We shall now study the kinematics of such collisions, restricting to those that results in the production of two products particles only, but it can be easily generalized. For the sake of compactness, we shall use a system of unit where the light velocity c would be taken as unity, i.e., e=1 So that mc^2 would be denoted by the mass m, the energy pc by momentum p, etc.

Let us take the reaction: $A_1+ A_2\rightarrow A_3+A_4$ ---------------------------(1)

The conservation principle \of energy and momentum dictate that

$$W_1+W_2 =W_3+W_4 \qquad \text{----------------------------(2)}$$

$$\text{and}\quad p_1+p_2=p_3+p_4 \quad \text{-------------------------------(3)}$$

where W's represent total energies and p's the momenta of the particles

Using relativistic relation for different particles, we get

$$W_n^2 = p_n^2 + m_n^2 \qquad n=1, 2, 3\ldots\ldots \quad \text{-----------------(4)}$$

From relativity, again, the total energy W and the components of momentum vector $\bar{p}$ constitute a four factor p_μ =(W,p), and the scalar product p_μ p_μ in 4-space is an invariant at a particular time.

$$p_\mu\, p_\mu = W_2 - p_2 = \text{invariant} \qquad \text{-----------------------------(5)}$$

p_2 being the square of 3d-momentum vector.

For a system of particles, in initial or final state,

$$p\mu\, p\mu = (\textstyle\sum W_n)^2 - (\sum p_n)^2 = \text{invariant} \quad \text{-------------------------(6)}$$

In reference to the relation (1), therefore, we have

$$(p\mu\, p\mu)\ \text{initial} = (W_1+W_2)^2 - (p_1+p_2)^2$$

$$= (\sqrt{p_1^2 - m_1^2} + \sqrt{p_2^2 + m_2^2}\,)^2 - (p_1 + p_2)^2 \quad \text{-------------(7)}$$

In Lab-frame (L-frame), assuming the target A_2 to be at rest, p^2=0. The kinetic energy T_1 of the incident particle A1 is then

$$T_1 = W_1 - m_1 = \sqrt{p_1^2 - m_1^2} - m_1$$

From (2), we therefore obtain directly

$$(p_\mu p_\mu)_{\text{initial}} = (\sqrt{p_1^2 - m_1^2} - m_2)^2 - p_1^2$$

$$= m_1^2 + m_2^2 + 2m_2\sqrt{p_1^2 + m_1^2}$$

$$= m_1^2 + m_2^2 + 2m_2(m_1+T_1)$$

$$= (m_1+m_2)^2 + 2m_2T_1 \qquad \text{-------------------------(8)}$$

Threshold energy: All the threshold, A_3 and A_4 are produced with zero momentum, i.e $p_3=p_4=0$ so, using (8)

$$(p_\mu p_\mu)_{\text{initial}} = (m_3+m_4)^2 \qquad \text{----------------------(9)}$$

Substituting $m_1=m_1+m_2$ and $m_f=m_3+m_4$ and explaining the invariance, we get from equ (8) and (9).

$$m_1^2 + 2m_2T_1 = m_f^2$$

$$= \quad T_1 = (m_f^2 - m_i^2)/2m_2 \qquad \text{-------------------------------(10)}$$

Again, the Q-value of the reaction is

$$Q=m_1+m_2-m_3-m_4= m_i-m_f \qquad \text{---------------------------(11)}$$

Threshold energy, $E_{th} =T_1= \frac{(m_f+m_i)(m_f-m_i)}{2m_2}$ using equ (10)

$$= - (Q/2m_2)\,(m_i+m_f)$$

$$= (Q/2m_2)\,(Q-2m_i) \text{ using equ (11)}$$

$$E_{th}= Q[Q/2m_2- (m_1+m_2)/m_2] \qquad \text{-----------------------(12)}$$

Non-relativistic collision: Here $Q\leq m_1$ (or m_2). So from equ (12), we obtain

$$E_{th}= -Q\frac{m_1+m_2}{m_2} \qquad \text{---------------------------------------(13)}$$

Extreme relativistic case: Here $Q\geq m_1$ and also m_2 so that

$$E_{th}= Q^2/2m_2 \;= E_{th} \propto Q^2 \qquad \text{----------------------------(14)}$$

$$Q/E_{th}= 2m_2/Q \;\leq 1$$

This implies that only a small fraction of E_{th} goes to provide the reaction energy.

14.8 Let us sum up

- Time reversal operator is defined as that operator which reverses the direction of time, or the direction of all motions
- The invariance of CPT transformation implies that if an interaction is not invariant under any one of the C, P or T-operations.
- The electric dipole moment of a nucleus in its ground state vanishes. This is also true for all non-degenerate excited states of the nucleus.
- Many elementary particles are the product of high energy (relativistic) collisions.

14.9 Review questions:

1. What is time reversal? Explain

2. What is meant by eight-fold way or octet symmetry? Explain

3. Explain- CPT invariance?

4. What is relativistic kinetics of elementary particles?

14.10 Further readings

1. Nuclear Physics- Irving Kaplan, Oxford & I.B.H Pub&Co.

2. Nuclear Physics- R,.R.Roy and B.P.Nigam, John Wiley 1967

Distance Education- CBCS- (2018-19 Academic year onwards)

M.Sc (PHYSICS) Degree Examination

Question paper pattern (ESE)

NUCLEAR PHYSICS 34542

Time: 3 hours **Maximum: 75 Marks**

PART- A (10×2=20 Marks)

Answer all questions

1. Give the selection rules for gamma transition.
2. What is meant by alpha decay?
3. Write a note on Schmidt lines.
4. What are tensor forces?
5. What is partial wave analysis?
6. Define thermal neutrons.
7. What is meant by thermo nuclear reactor?
8. Write a note on neutrino.
9. State- Mesons
10. Write Gel-Mann-Nishijima formula.

PART-B (5×5=25 Marks)

Answer all questions choosing either (a) or (b)

11. a. How is internal conversion co-efficient of gamma rays obtained? Explain

    (OR)

    b. Give an account on parity violation in β- decay.
12. a. Give the theory of nuclear quadrupole moment.

    (OR)

    b. Explain how spin-orbit coupling can be accounted on the basis of shell model.
13. a. Give the theory of a compound nucleus formation and its decay.

    (OR)

    b. How is deuteron wave function normalized?
14. a. Explain neutron cycle in a nuclear reactor. Derive four factor formula.

    (OR)

    b. Write a detail note on sources of stellar energy.
15. a. Define strangeness. Explain

    (OR)

    b. Explain SU(2) and SU(3) symmetry groups.

PART-C (3×10=30 Marks)

Answer any THREE questions:

16. Discuss the Gamow's theory of α-decay.
17. Give Fermi's theory of β-decay.
18. Give the simple theory of deuteron.
19. Obtain Breit-Wigner one level formula for resonance scattering. Deduce the level width.
20. What are the fundamental interaction? Explain